The Uncertainty Channel of Monetary Policy

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Abstract

I propose a novel channel of monetary policy – *the uncertainty channel* – that works opposite of the well-known signaling channel by inducing uncertainty about economic shocks and altering their macroeconomic effects. In a workhorse New Keynesian framework with information frictions, I show that the central bank can effectively utilize this channel for macroeconomic stabilization by decreasing the informativeness of its actions via deviations from the policy rule. Thus, the optimal monetary policy is characterized by a policy rule that features policy shocks with a uniquely optimal variance. I also study the effects of monetary shocks in times of high versus low policy uncertainty in the US economy, and present empirical evidence consistent with model predictions.

Keywords: New Keynesian model, information frictions, monetary policy shocks, optimal monetary policy

JEL Classification Codes: D82, D83, E52, E58

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Introduction

Monetary policy rates are empirically well characterized by an underlying policy rule that responds to inflation and the output gap, along with frequent deviations from the rule-based rate. In a standard New Keynesian model, these deviations take the form of policy shocks that generate economic fluctuations and reduce social welfare. Therefore, a central bank would seek to minimize such deviations to eliminate macroeconomic volatility due to policy shocks.

However, under incomplete information, the effects of policy shocks on the economy may be altered by various factors. In particular, if the private sector does not directly observe the shocks driving the economy, an increase in the policy rate may be perceived as a policy shock or a policy response to another fundamental shock. As a result, policy shocks as well as other disturbances will have different effects on the economy under incomplete information. Moreover, the economic agents' subjective uncertainty about the realization of economic shocks, including policy innovations, may change how the economy responds to those disturbances. Hence, the relative volatilities of policy and fundamental shocks alter the effects of those shocks on the economy.

This phenomenon is due to a novel channel of monetary policy, which I refer to as the *uncertainty channel*. The latter works opposite of the well-known signaling channel by reducing the informativeness of the central bank's actions and increasing uncertainty about the realization of economic disturbances. For the central bank, this channel is particularly useful for reducing the economic impact of inefficient shocks such as markup shocks, which, unlike demand or productivity shocks, cannot be conventionally neutralized by the interest rate channel. Hence, the central bank can utilize the uncertainty channel to its advantage to reduce the macroeconomic volatility generated by such inefficient shocks.

Although the signaling and uncertainty channels interact closely, their working mechanisms are distinct. The signaling channel works dynamically in each period providing information about the current state of the economy via the policy rate. In contrast, the uncertainty channel is a static mechanism embedded in the policy rule that works through the time-invariant volatility of policy shocks to reduce the precision of the private sector's beliefs about the economy.

On the other hand, the signaling effects critically depend on the co-movement of the policy rate with the underlying shocks. Therefore, the central bank's response to inflation and the output gap determines the above-mentioned co-movements and therefore the extent of policy signaling. As a result, the central bank cannot independently control its signaling effects without altering the policy rule, similar to the Tinbergen (1952) principle of one instrument per target. However, the central bank can use the uncertainty channel to reduce the signaling effects of policy actions without changing the policy rule.

The paper makes two main contributions. First, I show that the economic effects of exogenous disturbances depend on the volatility of monetary policy shocks relative to the volatility of the fundamental shocks. In particular, under high volatility of policy shocks, the effects of both markup and monetary policy shocks resemble those of monetary policy shocks in a standard New Keynesian model. Conversely, under low volatility of policy shocks, the effects of markup and monetary policy shocks are similar to those of markup shocks typically found in the New Keynesian literature. I also test these implications for the US economy and find empirical evidence consistent with model predictions.

Second, I show that the central bank may utilize the uncertainty channel to mitigate the adverse effects of inefficient shocks. Specifically, the uncertainty channel reduces the signaling effects of policy actions, increases the uncertainty about fundamental shocks, and reduces their economic impact. Thus, central banks can use information frictions to their advantage for the optimal monetary policy design. Contrary to the conventional wisdom, I show that the optimal policy rule is characterized by monetary policy shocks with a positive variance. Most importantly, the gains from the uncertainty channel outweigh the welfare cost of policy shocks, and there exists a uniquely optimal variance of policy shocks that maximizes social welfare. To the best of my knowledge, this is the first paper to study the potential welfare-improving effects of policy shocks.

It is important to note, however, that the welfare-improving effects of deviations from policy rules are due to the presence and possibility of these deviations, rather than their realizations. Monetary policy shocks are inefficient disturbances themselves and reduce welfare. Nevertheless, in this context, the uncertainty surrounding the realization of these shocks dampens their impact on the economy, as argued by Angeletos and Pavan (2007) and Angeletos et al. (2016). Therefore, although the monetary policy shock itself is an inefficient disturbance, its presence in the policy rule has net welfare-improving effects because it increases uncertainty about those shocks.

The detrimental effects policy signaling have been also studied by Baeriswyl and Cornand (2010), which shows that in a static sticky-information economy, monetary policy signaling about inefficient markup shocks exacerbates their effects on the economy. On the other hand, Ou et al. (2022) finds that increased central bank transparency about efficient productivity shocks reduces social welfare in the presence of nominal rigidities. In contrast, I show that a central bank can use its informational advantage to reduce the precision of those shocks and alter their effects on the economy. Different from Baeriswyl and Cornand (2010), I study a sticky-price economy, where the central bank faces a trade-off between output gap and inflation, and show that the central bank can utilize the uncertainty channel to mitigate that trade-off. Then, I also explore the uncertainty channel in a dynamic New Keynesian model with a richer setup and discuss its implications for optimal monetary policy design.

The mechanism presented in the paper builds on the findings of Angeletos and Pavan (2007) and Angeletos et al. (2016) which study the welfare effects of inefficient shocks. In particular, they find that social welfare is decreasing in the precision of those shocks and that central banks have limited ability to counteract such shocks. On th other hand, Baeriswyl and Cornand (2010) argue that the central bank's communication about inefficient shocks amplifies the output gap induced by such disturbances. The mechanism proposed in this paper works in the opposite direction by increasing the uncertainty about those shocks. I show that, in addition to suppressing public information on inefficient shocks, the optimal policy requires the central bank to deviate from its policy rule in a systematic and unpredictable manner to decrease the informativeness of its actions. As a result, the uncertainty about those shocks increases, which in turn reduces their economic impact.

The framework developed in this paper also shares similarities with Angeletos and Lian (2018). The crucial difference is that in my model the private sector lacks information about the current state of the economy, as well as the future states. It is also close to Nimark (2008), in which imperfectly informed firms use their observations of aggregate variables to form beliefs about the economy. Additionally, in my model, the policy rate contains information about the likely realizations of shocks and is used by firms in their price-setting decisions. This mechanism is based on the informational advantage of the central bank over the private sector, which has been documented by Romer and Romer (2000) and Nakamura and Steinsson (2018) among others. I study the optimal monetary policy design under incomplete information when a central bank can exercise this advantage and make strategic use of its superior information.

This paper contributes to a large body of literature studying the price setting problem of firms in the presence of information frictions by Mankiw and Reis (2002), Woodford (2003a), Nimark (2008), Angeletos et al. (2016) motivated by the earlier contributions by Lucas (1972, 1973, 1975).

The paper also contributes to the vast literature on the signaling role of policy actions by Baeriswyl and Cornand (2010), Berkelmans (2011), Melosi (2016), Kohlhas (2022), Jia (2023) among others. In particular, novel in this literature, I show that the central bank can use information frictions to its advantage to reduce the precision of private sector beliefs about inefficient shocks and alter their effects on the economy.

This paper also speaks to the literature on optimal monetary policy under incomplete information by Adam (2007), Lorenzoni (2010), Berkelmans (2011), Angeletos and La'O

(2020), Iovino et al. (2022) among others, by illustrating the potential welfare-improving effects of deviations from policy rules. Finally, in line with the findings of Morris and Shin (2002), this paper sheds light on the role of public information about inefficient disturbances by demonstrating its detrimental effects on social welfare.

The rest of the paper is organized as follows. In Section 1, I present a simple static New Keynesian model to motivate and illustrate the main results of the paper. Then, in Section 2, I present a dynamic New Keynesian model with a richer setup and discuss its key features. Section 3 analyzes the signaling and uncertainty channels in the model economy, while Section 4 discusses the optimal monetary policy framework. Finally, Section 5 presents an empirical analysis of the effects of monetary shocks in times of high versus low policy uncertainty in the US economy.

1 A simple model

Consider a simple static New Keynesian model with a representative household, a continuum of firms, and a monetary authority. The economy lives for only one period.

1.1 Households

Households supply labor N to firms and consume C according to preferences given by

$$U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$$

subject to the following budget constraint

$$PC = WN + T$$

where P is the price level, W is the wage rate, and T are lump-sum transfers to the household, including profits from the firms.

The consumption aggregate C is a CES function of differentiated goods produced by firms, given by

$$C = \left(\int_j C_j^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

where ϵ is the elasticity of substitution between goods.

The optimal choice of labor and consumption for the household implies

$$C^{\sigma}N^{\varphi} = \frac{W}{P} \tag{1}$$

1.2 Firms

Monopolistically competitive firms produce differentiated goods using household labor. The production function of firm j is given by

$$Y_j = N_j$$

Assume that prices are sticky, such that a fraction θ of firms set prices to 1, while the remaining firms choose an optimal price P_j^* that maximizes their profits given by

$$\max_{P_j^*} \quad E_j\left[\Lambda\left(P_j^* Y_j - W N_j\right)\right]$$

subject to the demand constraint

$$Y_j = \left(\frac{P_j^*}{P}\right)^{-\epsilon} C$$

where $\Lambda = C^{-\sigma}$ is the household marginal utility of consumption and ϵ is the elasticity of substitution between goods.

The expectation operator $E_j\left[\,\cdot\,\right]$ above denotes the expectation of firm j conditional on its information set, i.e. $E_j\left[\,\cdot\,\right] \equiv E\left[\,\cdot\,|\,\mathcal{I}_j\right]$.

Profit maximization of firms implies

$$E_j\left[\Lambda Y_j\left(P_j^* - \mathcal{M}PMC\right)\right] = 0 \tag{2}$$

where $MC = W/P$ is the real marginal cost of production and $\mathcal{M}$ is the desired markup of firms.

Let's assume that the desired markup $\mathcal{M}$ is subject to a shock e^u such that

$$\log\left(\mathcal{M}\right) = \log\left(\overline{\mathcal{M}}\right) + e^u \qquad\qquad e^u \sim N(0, \sigma^2_{e^u})$$

where $\overline{\mathcal{M}}$ is the steady-state level of mark-ups. I assume that labor income is subsidized by lump-sum taxes to restore the efficient steady-state level of output. With the subsidy in place, the steady-state level of mark-ups is given by $\overline{\mathcal{M}} = 1$.[1]

1.3 Central bank

The central bank sets the nominal money supply M according to the rule

$$\log(M) = -\phi e^u - e^m \qquad\qquad e^m \sim N(0, \sigma^2_{e^m}) \tag{3}$$

where e^m is a monetary policy shock.[2]

Different from Mankiw and Reis (2002) and Woodford (2003a), which treat money supply as exogenous, I assume that the central bank responds to the markup shock e^u with parameter ϕ and may deviate from the implied rule to the extent of the monetary policy shock e^m. This specification is similar to the policy rules in New Keynesian models with a difference that in my setup the central bank responds to the markup shocks rather than prices.[3] I use this specification in the simple model to get an analytical solution of the model and illustrate the main results of the paper. Then, I consider typical interest rate rules responding to inflation and the output gap in the dynamic model in Section 2.

The policy rule above also features a monetary policy shock e^m, representing deviations from the rule. The role of the monetary policy shock, as will become clear later, is to introduce uncertainty about the realization of the markup shock. The central bank may use this uncertainty to its advantage to reduce the impact of the markup shock on the economy.

Although the assumed monetary policy rule takes the form of a simple policy rule, it also nests the otherwise optimal state-contingent policy rule. In particular, since the model is linear, the optimal money supply must be a linear function of the state, i.e. the markup shock e^u. Then, in addition to the optimal response to the markup shock, I also introduce a policy shock e^m that may mute the signaling effects of monetary policy and potentially improve the welfare of the economy.

Finally, markets clear with

$$C = Y \qquad\qquad M = PY \qquad\qquad N = \int_j N_j \, dj \tag{4}$$

such that product and labor markets clear and the the money supply determines the aggre-

[1] See Galí (2015) for a discussion of the distortion of the efficient steady state due to monopolistic competition.

[2] I assume that the shock enters the policy rule with a negative sign, such that a positive shock decreases the money supply, to be consistent with the contractionary effects of positive shocks in interest rate rules.

[3] Commonly used simple policy rules include Taylor type rules for money supply or interest rates that respond to inflation and the output gap. However, since the latter are functions of the state variables, the policy rules may also be expressed in terms of the state variables directly.

gate demand in the economy, as in Woodford (2003a).

1.4 Information structure and equilibrium

Firms have limited knowledge about the state of the economy. They observe noisy signals but do not observe e^u or e^m directly. Specifically, each firm j receives two private and public signals given by

$$\begin{aligned} S_j^u &= e^u + \zeta_j^u \qquad\qquad \zeta_j^u \sim \mathcal{N}\left(0, \sigma_{\zeta^u}^2\right) \\ S_j^m &= m \end{aligned}$$

where $m \equiv \log(M) = -\phi e^u - e^m$ is the money supply. Therefore, the information set of firm j is given by $\mathcal{I}_j = \left\{S_j^u, S_j^m\right\}$.

The information structure of the model is similar to the one in Morris and Shin (2002), where agents receive noisy private and public signals about some underlying fundamental. In my model, the markup shock e^u is the fundamental, while the money supply m is the public signal and the monetary policy shock e^m is the noise in the public signal. However, different from Morris and Shin (2002), the money supply not only takes the role of a public signal but is also a policy instrument and determines the aggregate demand in the economy. I discuss this feature in more detail in Section 1.6.

The economy exhibits two stages. In the first stage, each firm j observes the signals S_j^u, S_j^m and sets its price P_j^* according to Eq. (2). In the second stage, each firm j hires labor N_j and meets the demand for its product Y_j at the price P_j^*. In log-linear form, the optimal price of firm j is given by

$$p_j^* = E_j\left[p + mc + e^u\right]$$

which, using the monetary policy rule, household optimality, and market clearing conditions in Eqs. (1), (3) and (4), can be rewritten as

$$p_j^* = (1-\theta)(1-\sigma-\varphi) E_j\left[p^*\right] + (\sigma+\varphi) E_j[m] + E_j\left[e^u\right]$$

where $(1-\theta)(1-\sigma-\varphi)$ stands for the degree of *strategic complementarity/substitutability* in the model in the terminology of Mankiw and Reis (2002) and Woodford (2003b). If $\sigma+\varphi<1$, then firms' pricing decisions are strategic complements, while if $\sigma+\varphi>1$, then firms' pricing decisions are strategic substitutes.[4] As we will see later, the degree of strategic complementarity/substitutability in the model does not affect the main results of the paper.

Definition 1 (Equilibrium)

For realizations of exogenous disturbances e^u and e^m, competitive equilibrium consists of prices P_j^*, P_j, P, W and allocations Y_j, N_j, Y, N, C, M, such that

- *the representative household chooses consumption C and labor N according to the optimality conditions in Eq. (1)*
- *each firm j sets its price P_j^* according to Eq. (2) given its information set $\mathcal{I}_j$, then hires labor N_j and meets demand Y_j at the price P_j^**
- *the central bank sets the money supply M according to the rule given in Eq. (3)*
- *markets clear according to conditions in Eq. (4)*

Before characterizing the general equilibrium in the model, it is useful to define a few notations that are widely used throughout this paper. For a variable x, let

$$\overline{E}^{(1)}\left[x\right] = \int_j E_j\left[x\right]\, dj \qquad\qquad \overline{E}^{(k)}\left[x\right] = \int_j E_j\left[\overline{E}^{(k-1)}\left[x\right]\right]\, dj \tag{5}$$

denote the k-th order beliefs of firms about x. Specifically, $\overline{E}^{(1)}\left[x\right]$ is the average belief of x across firms, while $\overline{E}^{(k)}\left[x\right]$ is the average belief of $(k-1)$-th order belief of firms about x.

The general equilibrium of the model is characterized by the following system of equations

$$\begin{aligned}
y &= -p - \phi e^u - e^m \\
p &= (\sigma + \varphi)\sum_{k=1}^{\infty}(1-\theta)^k\,\overline{E}^{(k)}\left[y\right] + \sum_{k=1}^{\infty}(1-\theta)^k\,\overline{E}^{(k)}\left[e^u\right]
\end{aligned}$$

where $y = \log(Y)$ and $p = \log(P)$.[5]

Equilibrium in the model is characterized by the two equations above, which are essentially the aggregate demand and the aggregate supply or the Phillips curve, respectively. Unlike in the standard New Keynesian model, in this model, the Phillips curve features firms' higher-order beliefs about the output and the markup shock.

[4]See Woodford (2003b) for a detailed discussion of the strategic complementarity and substitutability of pricing actions in the New Keynesian models, as well as empirical evidence on the degree of strategic complementarity or substitutability in the literature.

[5]The derivation of the equilibrium system is available in the Online Appendix.

We can solve the model by the method of undetermined coefficients. I guess and verify that the optimal prices set by firms are linear functions of the signals S_j^u and S_j^m, i.e.

$$p_j^* = \alpha_1 S_j^u + \alpha_2 S_j^m$$

where α_1 and α_2 are the coefficients to be determined. Then, I substitute the optimal prices into the aggregate demand and aggregate supply equations and solve for the coefficients α_1 and α_2.[6] The solution is given by

$$z = \Gamma e \tag{6}$$

where $z = [y, p]'$, $e = [e^u, e^m]'$ and

$$\Gamma = \begin{bmatrix} -\left(\alpha_1 (1-\theta) + \phi \left(1 - \alpha_2 (1-\theta)\right)\right) & -\left(1 - \alpha_2 (1-\theta)\right) \\ \alpha_1 (1-\theta) - \phi \alpha_2 (1-\theta) & -\alpha_2 (1-\theta) \end{bmatrix} \tag{7}$$

with

$$\alpha_1 = \sigma_{e^m}^2 \sigma_{e^u}^2 \Psi^{-1} \qquad \alpha_2 = \left((\sigma + \varphi) \Psi - \phi \sigma_{e^u}^2 \sigma_{\zeta^u}^2\right) \Omega^{-1} \Psi^{-1} \tag{8}$$

where $\Omega = \theta + (1-\theta)(\sigma + \varphi)$ and $\Psi = \phi^2 \sigma_{e^u}^2 \sigma_{\zeta^u}^2 + \sigma_{e^m}^2 \left(\Omega \sigma_{e^u}^2 + \sigma_{\zeta^u}^2\right)$.

1.5 Complete information benchmark

Under complete information, firms have full knowledge about the exogenous shocks e^u and e^m. Thus, we have $\overline{E}^{(k)}[x] = x$ for all k for any x.

Then, the equilibrium system simplifies to

$$y = -p - \phi e^u - e^m \tag{AD}$$

$$p = (\sigma + \varphi) \frac{1-\theta}{\theta} y + \frac{1-\theta}{\theta} e^u \tag{AS}$$

The equilibrium in the economy is characterized by the aggregate demand (AD) and the aggregate supply (AS) equations above. Solving for y and p, we get

$$z = \Gamma_c e \tag{9}$$

[6] The details of the solution method are available in the Online Appendix.

where $z = [y, p]'$, $e = [e^u, e^m]'$ and

$$\Gamma_c = \Omega^{-1} \begin{bmatrix} -(1-\theta+\phi\theta) & -\theta \\ (1-\theta)(1-\phi(\sigma+\varphi)) & -(1-\theta)(\sigma+\varphi) \end{bmatrix} \tag{10}$$

where $\Omega = \theta + (1-\theta)(\sigma+\varphi)$.

1.6 The uncertainty channel

To study the role of information frictions in the model, and illustrate the main results of the paper, first, let's assume that firms observe only the public signal on money supply m, but not the private signal. This is the special case of the model where $\sigma^2_{\zeta^u} \to \infty$, such that the private signal is uninformative about the markup shock e^u. Then, conditional on observing the money supply m, firms' beliefs are given by

$$E_j[e^u] = -\frac{\phi\sigma^2_{e^u}}{\phi^2\sigma^2_{e^u} + \sigma^2_{e^m}} m \qquad E_j[e^m] = -\frac{\sigma^2_{e^m}}{\phi^2\sigma^2_{e^u} + \sigma^2_{e^m}} m \tag{11}$$

As we can see from the expressions above, the beliefs of firm j are linear functions of the money supply m with loadings that depend on the parameters of the model. Most importantly, these relationships depend on the policy parameters ϕ and $\sigma^2_{e^m}$, which determine the precision of the beliefs of firm j. These parameters affect the belief formation via their effect on the co-movements between the shocks and the money supply. Specifically, the beliefs of firm j about the shocks can be expressed as

$$E_j[e^u] = \frac{\text{Cov}(e^u, m)}{\text{Var}(m)} m \qquad E_j[e^m] = \frac{\text{Cov}(e^m, m)}{\text{Var}(m)} m$$

Since the central bank responds to the markup shock e^u with parameter ϕ, the money supply m is informative about the markup shock. This feature of the model is essentially the signaling channel of monetary policy studied by Melosi (2016). However, the money supply is also affected by the monetary policy shock e^m, which adds noise to the money supply and obscures the signal about the markup shock. This feature, in turn, is what I refer to as the *uncertainty channel* of monetary policy, working opposite of the signaling channel.

To see how the presence of a monetary shock alters firms' beliefs on the markup shock, let's substitute the policy rule in Eq. (3) for the money supply m. Then, the first-order beliefs in case of a markup shock are given by

$$\overline{E}^{(1)}\left[e^u\right]\Bigg|_{e^u}=\frac{\phi^2\sigma_{e^u}^2}{\phi^2\sigma_{e^u}^2+\sigma_{e^m}^2}e^u \qquad \overline{E}^{(1)}\left[e^m\right]\Bigg|_{e^u}=\frac{\phi\sigma_{e^m}^2}{\phi^2\sigma_{e^u}^2+\sigma_{e^m}^2}e^u \tag{12}$$

We can see from the above, that because of the presence of the monetary shock, the first-order belief of firms on markup shock is only a fraction of the true shock. This is due to the fact that a decrease in money supply may be a sign of an inflationary markup shock or equivalently a contractionary monetary shock. As a result, conditional on the signal, the beliefs of an average firm are split between the markup and monetary shocks, as given by Eq. (12).

Similar to markup shock, firms' beliefs after a true monetary shock are split between the two shocks. In particular, a monetary shock forms the following beliefs about the state of the economy

$$\overline{E}^{(1)}\left[e^u\right]\Bigg|_{e^m}=\frac{\phi\sigma_{e^u}^2}{\phi^2\sigma_{e^u}^2+\sigma_{e^m}^2}e^m \qquad \overline{E}^{(1)}\left[e^m\right]\Bigg|_{e^m}=\frac{\sigma_{e^m}^2}{\phi^2\sigma_{e^u}^2+\sigma_{e^m}^2}e^m \tag{13}$$

Due to the strategic uncertainty between the markup and monetary shocks, conditional on the money supply signal, firms' beliefs are split between both shocks. As a result, the response of the economy to both shocks becomes a convex combination of the responses under complete information. To see the mechanism behind this result, we can note from Eq. (8) that with $\sigma_{\zeta^u}^2\to\infty$, we have $\alpha_1=0$ and

$$\alpha_2=\frac{\phi\sigma_{e^u}^2\left(\phi\left(\sigma+\varphi\right)-1\right)+\sigma_{e^m}^2\left(\sigma+\varphi\right)}{\Omega\left(\phi^2\sigma_{e^u}^2+\sigma_{e^m}^2\right)}$$

Hence, we can express α_2 as

$$\alpha_2=\kappa\alpha_u\phi^{-1}+\left(1-\kappa\right)\alpha_m$$

where $\alpha_u=\left(\phi\left(\sigma+\varphi\right)-1\right)\Omega^{-1}$, $\alpha_m=\left(\sigma+\varphi\right)\Omega^{-1}$ and $\kappa=\frac{\phi^2\sigma_{e^u}^2}{\phi^2\sigma_{e^u}^2+\sigma_{e^m}^2}$ is the proportion of money supply volatility due to the markup shock.

We can note from Eq. (9) that α_u and α_m are the responses of the optimal prices to negative markup and monetary shocks under complete information.[7] Thus, α_2 is a convex

[7]We can also see that α_u and α_m may be interpreted as

$$\alpha_u=\phi\lim_{\sigma_{e^m}^2\to 0}\alpha_2 \qquad \alpha_m=\lim_{\sigma_{e^m}^2\to\infty}\alpha_2$$

i.e. the responses to the markup and monetary shocks under the limiting cases of $\sigma_{e^m}^2\to 0$ and $\sigma_{e^m}^2\to\infty$.

combination of $\alpha_u \phi^{-1}$ and α_m, i.e. the responses to the markup and monetary shocks under complete information, weighted by the proportion of money supply volatility due to each shock. Therefore, under incomplete information, the response of optimal prices to a unit increase in money supply is a weighted average of the responses to the markup shock of $-1/\phi$ and that of the monetary shock of -1 (i.e. the shocks that would generate the observed unit increase in money supply).

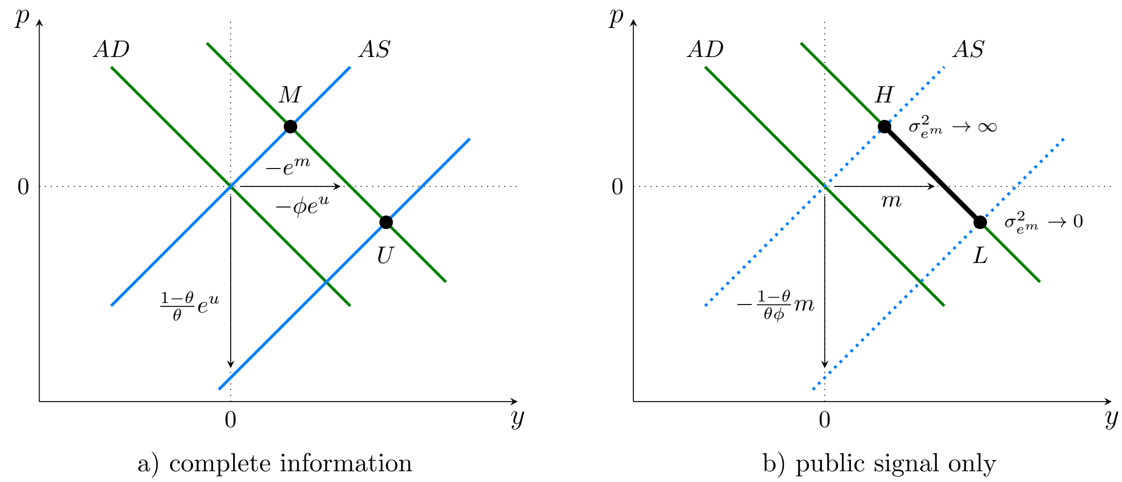
We can also see that with $\sigma^2_{\zeta^u} \to \infty$, Eqs. (6) and (7) reduce to

$$\begin{bmatrix} y \\ p \end{bmatrix} = \begin{bmatrix} 1 - \alpha_2 (1 - \theta) \\ \alpha_2 (1 - \theta) \end{bmatrix} m \tag{14}$$

Thus, when the money supply is the only signal available to the firms, it becomes the only state variable that drives the economy. This comes from its dual role as both the only signal observed by the firms and the measure of nominal spending. As a result, regardless of the true realization of the shocks, both markup and monetary shocks have identical effects on the economy, up to an appropriate scale (i.e. the effects of a 1 unit monetary shock are identical to those of a $1/\phi$ units of markup shock).

We can also see the effects of money supply (i.e. those of markup and monetary shocks behind the scenes) in the AD-AS framework presented in Fig. 1. Under complete information (the left panel of Fig. 1), an expansionary monetary shock of $e^m < 0$ shits the AD curve to the right by $-e^m$, while the AS curve is unaffected. Hence, the equilibrium in the economy is at point M with higher output and prices relative to the steady state. However, a deflationary markup shock of $e^u < 0$ (such that it generates the same increase in money supply as e^m), lowers the AS curve by $\frac{1-\theta}{\theta} e^u$, while the AD curve shifts to the right by $-\phi e^u$ due to the central bank's reaction to the shock. As a result, the economy settles at point U with higher output and lower prices.

In case of incomplete information with only public signal on money supply (the right panel of Fig. 1), the only relevant variable is the money supply regardless of the true shock. For a given money supply of m, equivalent to what would prevail as a result of markup or monetary shocks described above, the equilibrium in the economy is a convex combination of pints H and L (which are just the points M and U, but relabeled). The point H is the limiting equilibrium of the economy as $\sigma^2_{e^m} \to \infty$, i.e. high volatility of monetary shocks. In this case, monetary shocks are the dominant sources of fluctuations, and realizations of both shocks are perceived as monetary shocks. Similarly, point L is the limiting equilibrium of the economy as $\sigma^2_{e^m} \to 0$, i.e. when monetary shocks have infinitesimal volatility. In this case, markup shocks are the main drives of economic fluctuations and realizations of both shocks are perceived as markup shocks.



Note: The figure plots the effects of a money supply increase as a result of a monetary policy shock versus a policy response to a markup shock. In the first case, the money supply is given by $m = -e^m$, equal to $m = -\phi e^u$ in the second case.

Figure 1: Model equilibria under complete and incomplete information

So far we assumed that the money supply is the only signal received by firms for the sake of illustration of the uncertainty channel. Now, I generalize this result to the case of both private and public signals available to the firms.

Recall that with money supply as the only signal, firms' beliefs were split between the markup and monetary shocks. With both the private and public signals, firms' beliefs are similarly split between the two shocks, unless the precision of the private signal is very high. In this case, the private signal is perfectly informative about the markup shock, which, combined with the public signal, reveals the true state of the economy. However, if the private signal is noisy, firms' beliefs are split between the two shocks, which alters the effects of both shocks on the economy.

Proposition 1 below formalizes the results presented above. Under incomplete information with both private and public signals, the effects of both markup and monetary shocks on the economy are weighted averages of the corresponding effects under no monetary shock and a monetary shock with infinite volatility. The latter are in turn linear combinations of the effects of the two shocks under complete information.

Proposition 1 (The effects of shocks under incomplete information)

Let

$$\Gamma_0 \equiv \lim_{\sigma^2_{em} \to 0} \Gamma \qquad \Gamma_\infty \equiv \lim_{\sigma^2_{em} \to \infty} \Gamma$$

such that Γ_0 *and* Γ_∞ *are the effects of the shocks with no monetary shock and infinite volatility of the monetary shock, respectively.*

Then, the effects of the shocks under incomplete information are given by

$$\Gamma = \kappa\Gamma_0 + (1-\kappa)\Gamma_\infty$$

where $\kappa = \frac{\phi^2\sigma^2_{e^u}\sigma^2_{\zeta^u}}{\phi^2\sigma^2_{e^u}\sigma^2_{\zeta^u}+\sigma^2_{e^m}\left(\Omega\sigma^2_{e^u}+\sigma^2_{\zeta^u}\right)}$ *is the weight of the effects under no monetary shock, and*

$$\Gamma_0 = \Gamma_c H_0 \qquad\qquad \Gamma_\infty = \Gamma_c H_\infty$$

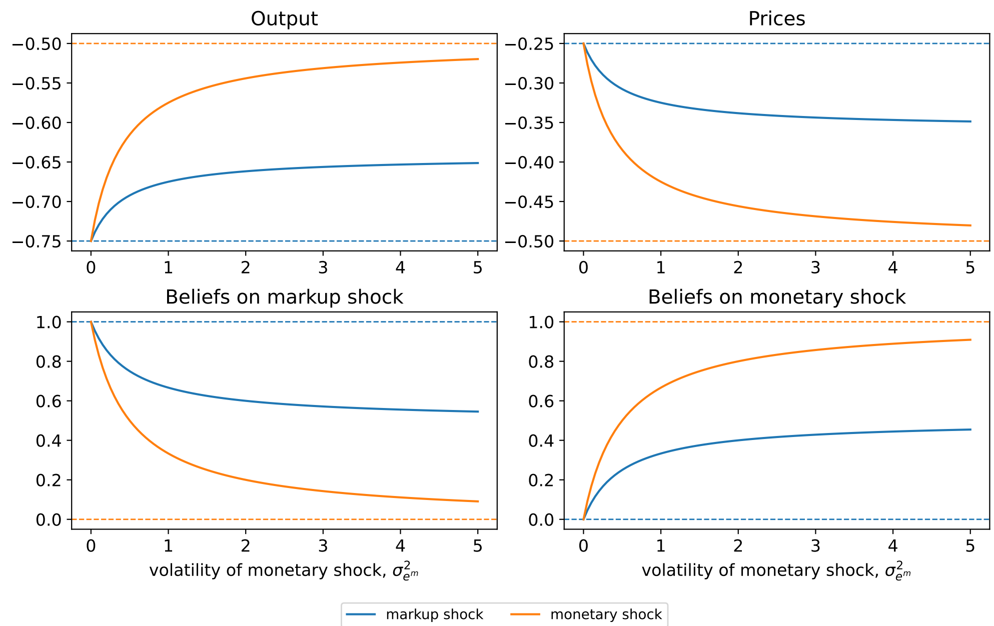
are linear combinations of the effects under complete information with

$$H_0 = \begin{bmatrix} 1 & 1/\phi \\ 0 & 0 \end{bmatrix} \qquad\qquad H_\infty = \begin{bmatrix} \frac{\Omega\sigma^2_{e^u}}{\Omega\sigma^2_{e^u}+\sigma^2_{\zeta^u}} & 0 \\ \frac{\phi\sigma^2_{\zeta^u}}{\Omega\sigma^2_{e^u}+\sigma^2_{\zeta^u}} & 1 \end{bmatrix}$$

These implications of Proposition 1 are plotted in Fig. 2. The figure shows the effects of markup and monetary shocks on output and prices under incomplete information, i.e. the elements of the matrix Γ, given by

$$\gamma^u_y \equiv \frac{\partial y}{\partial e^u} \qquad\qquad \gamma^m_y \equiv \frac{\partial y}{\partial e^m} \qquad\qquad \gamma^u_p \equiv \frac{\partial p}{\partial e^u} \qquad\qquad \gamma^m_p \equiv \frac{\partial p}{\partial e^m}$$

Fig. 2 also plots the first order firm beliefs, i.e. $\overline{E}^{(1)}\left[e^u\right]$ and $\overline{E}^{(1)}\left[e^m\right]$, conditional on the realization of the shocks. For reference, the dashed lines represent the effects of the shocks under complete information.



Note: The dashed lines represent the effects of the shocks under complete information, while the solid lines represent the effects under incomplete information. The parameters are set to $\sigma = 1$, $\varphi = 1$ $\theta = 2/3$, $\phi = 1$ and $\sigma^2_{e^u} = 1$, $\sigma^2_{\zeta^u} = 1$.

Figure 2: The effects of markup and monetary shocks under incomplete information

As we can see from the figure, the effects of both markup and monetary shocks on the economy depend on the volatility of the monetary policy shock $\sigma^2_{e^m}$. First, let's consider the effects of the shocks in the limiting case of $\sigma^2_{e^m} \to 0$. In this case, the markup shocks are the dominant source of fluctuations in the economy, hence firms attribute the observed changes in the money supply to the markup shock, regardless of the true shock. As a result, realizations of both shocks are perceived as markup shocks, and the effects of the shocks on the economy are identical to the effects of markup shocks under complete information.

On the other hand, as the volatility of the monetary policy shock increases with $\sigma^2_{e^m} \to \infty$, the effects of the monetary shock on the economy dominate. In this case, true monetary shocks are perceived as monetary shocks, and the effects of the shocks on the economy become similar to those of monetary shocks under complete information. However, since firms also receive a private signal about the markup shock, a true markup shock forms beliefs about both shocks and the effects of the markup shock on the economy become a weighted average of the effects of the two shocks.

We can also note that the effects of both markup and monetary shocks on the output are increasing, while the effects on prices are decreasing in $\sigma^2_{e^m}$. Proposition 2 below formalizes these results.

Proposition 2 (Volatility of the monetary shocks and the effects of the shocks) *Under incomplete information, the effects of both markup and monetary shocks on output and prices are altered by the volatility of the monetary policy shock $\sigma^2_{e^m}$.*

In particular

- *the effects of a markup shock on output are increasing, while the effects on prices are decreasing in $\sigma^2_{e^m}$*

$$\frac{\partial}{\partial \sigma^2_{e^m}}\left(\frac{\partial y}{\partial e^u}\right) > 0 \qquad\qquad \frac{\partial}{\partial \sigma^2_{e^m}}\left(\frac{\partial p}{\partial e^u}\right) < 0$$

- *the effects of a monetary shock on output are increasing, while the effects on prices are decreasing in $\sigma^2_{e^m}$*

$$\frac{\partial}{\partial \sigma^2_{e^m}}\left(\frac{\partial y}{\partial e^m}\right) > 0 \qquad\qquad \frac{\partial}{\partial \sigma^2_{e^m}}\left(\frac{\partial p}{\partial e^m}\right) < 0$$

The results of Proposition 2 have important implications for understanding the effects of markup and monetary shocks on the economy. In particular, the effects of the shocks on the economy depend on the volatility of the monetary policy shock. As the volatility of the monetary policy shock increases, it becomes the dominant source of fluctuations in the economy, and the effects of both shocks resemble those of monetary shocks under complete information. On the other hand, as the volatility of the monetary policy shock decreases, the effects of the shocks on the economy become similar to those of markup shocks under complete information.

These results also provide several testable implications for the effects of markup and monetary shocks on the economy. In Section 5, I test these implications using data on the US economy and find supporting evidence for the model's predictions.

1.7 Welfare

The central bank seeks to maximize the welfare of the representative household by choosing the optimal policy design. In particular, the central bank may choose the response parameter ϕ as well as the variance of the monetary policy shock $\sigma^2_{e^m}$.

To evaluate the welfare implications of the central bank's policies, I use a second-order approximation of the household welfare and express it in terms of welfare losses given by the following expression.[8]

$$\mathbb{L} = \frac{1}{2}\left(\left(\sigma+\varphi\right)E\left[y^2\right] + \frac{\epsilon\theta}{1-\theta}E\left[p^2\right]\right) + \frac{\epsilon\left(1-\theta\right)}{2}\alpha_1^2\sigma_{\zeta^u}^2$$

The welfare loss function L differs from the standard loss function in the New Keynesian models by the presence of the last term. Sticky prices and the resulting price dispersion in the economy are well-known sources of welfare losses in the New Keynesian models. However, in the presence of information frictions, the optimal prices set by firms differ due to the differences in their information sets. Since the private signals contain idiosyncratic noise, the optimal prices set by firms are dispersed around the aggregate optimal price. This dispersion in prices is an additional source of welfare losses for the households.

The loss function also has certain implications for the role of the public signals that contrast with Morris and Shin (2002) model. In the latter, the objective function of each agent is defined ad hoc as the quadratic loss function of the difference between the true state of the economy and the agent's actions. In this case, the social welfare is maximized when the agents correctly estimate the underlying fundamental.

In contrast, in my model, firms' actions are based on profit maximization, while social welfare is defined as household welfare. With this misalignment of firms' objectives and social welfare, inefficient shocks generate negative externalities on the economy, since firms' profit maximization results in volatility of output and prices and hence welfare losses. Therefore, a welfare-maximizing central bank would seek to reduce the response of the economy to inefficient shocks and minimize the welfare losses generated by these shocks.

1.8 Optimal policy

First, let's discuss the optimal policy under complete information. In this case, firms have full knowledge about e^u and e^m. In this setting, the model becomes the static version of an otherwise standard New Keynesian economy with complete information.

Theorem 1 (Optimal policy under complete information)
Under complete information, the optimal policy is characterized by the following policy parameters:

- *the optimal variance of the monetary policy shock $\sigma_{e^m}^2$ is equal to zero.*

[8] The derivation of the welfare loss function is available in the Online Appendix.

- *the optimal policy response parameter* ϕ *is equal to*

$$\phi^* = \frac{(\epsilon - 1)(1-\theta)}{\epsilon(1-\theta)(\sigma+\varphi)+\theta}$$

Theorem 1 derives the optimal policy response parameter that balances the adverse welfare effects of cost-push shocks on volatility of output and prices. Unlike demand-driven disturbances, cost-push shocks break the "divine coincidence" between the output gap and inflation and render their joint stabilization infeasible. Hence, the first-best policy minimizes their welfare effects by achieving the optimal trade-off between the output gap and inflation. On the other hand, under complete information, policy shocks are strictly welfare-reducing, thus the optimal volatility of these shocks is zero.

However, I show next that under incomplete information monetary policy shocks are not strictly welfare-reducing. In fact, apart from generating economic fluctuations, they dampen the effects of inefficient shocks on the economy and may have net welfare-improving effects. Hence, next, I study the optimal policy under incomplete information.

To illustrate the main mechanism behind the results of the paper, first I consider the incomplete information version of the model but with the public signal only. This is the limiting case of the full model as $\sigma^2_{\zeta^u} \to \infty$, i.e. the precision of the private signal goes to zero. The limiting case of the model is useful for illustrating the main mechanisms working in the model and analytically deriving the optimal policy parameters. Then, I consider the full-scale model with both private and public signals and characterize the optimal policy in that setting.

To better understand the welfare effects of the uncertainty channel, first, let's assume that the money supply is the only public signal that firms observe before setting their prices. Then, m becomes the state variable of the economy as both the money supply in the economy and the only signal available to firms. In this setting, the responses of the economy to either a monetary shock or a markup shock (that generates the same increase in the money supply) are identical.

Thus, to study the welfare effects of money supply fluctuations we can see from Eq. (14) that the volatility of output and prices are given by

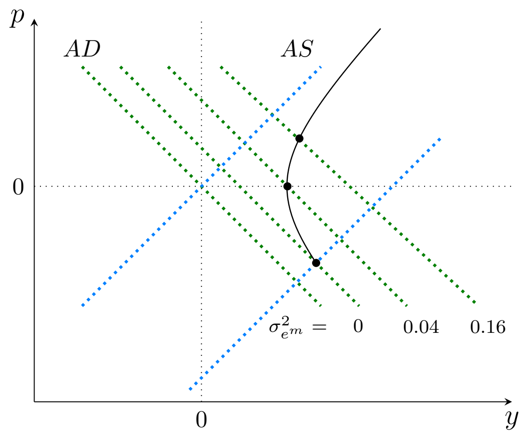
$$E\left[y^2\right] = \left(1-\alpha_2(1-\theta)\right)^2 E\left[m^2\right]$$
$$E\left[p^2\right] = \alpha_2^2(1-\theta)^2 E\left[m^2\right]$$

with $E\left[m^2\right] = \phi^2\sigma^2_{e^u} + \sigma^2_{e^m}$.

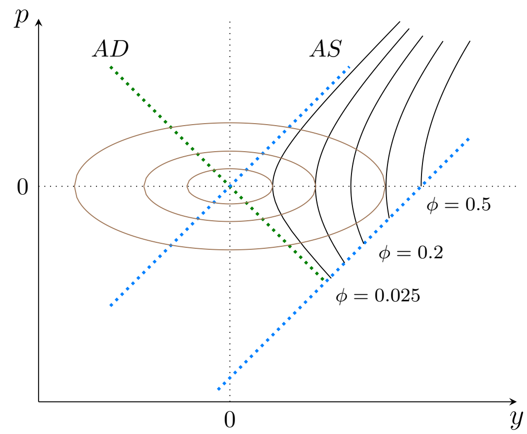
Hence, the welfare loss is

$$L = \frac{1}{2}\left(\left(\sigma+\varphi\right)\left(1-\alpha_2\left(1-\theta\right)\right)^2+\epsilon\theta\left(1-\theta\right)\alpha_2^2\right)E\left[m^2\right]$$

We can see that, under incomplete information, the volatility of monetary shock σ_{em}^2 plays a dual role in the welfare loss function. On the one hand, it directly affects the welfare loss by increasing the volatility of money supply and thus the volatility of output and prices. On the other hand, it affects the welfare loss indirectly by affecting firms' beliefs, as it affects firms' responsiveness to the money supply, given by α_2.



a) effects of σ_{em}^2 on model volatility



b) welfare contours and model volatility

Note: The figure plots the responses of output and prices to a 1 standard deviation increase in money supply as a function of the volatility of monetary policy shocks σ_{em}^2 and the policy response parameter ϕ.

Figure 3: Response of the economy to a 1 s.d. increase in money supply

Thus, to illustrate the effects of the monetary shock volatility on the welfare loss Fig. 3 plots the effects of a 1 standard deviation increase in the money supply on the economy. As we see in the left panel, with low volatility of policy shocks (e.g. when $\sigma_{em}^2 = 0.04$) an increase in the money supply results in a smaller shift in the AD curve compared with high volatility. However, due to the uncertainty channel, the equilibrium output increases, while prices remain unchanged, because of the ambiguity in the source of the money supply increase (i.e. inflationary monetary expansion versus deflationary markup shock).

On the other hand, if the volatility of policy shocks is high, an increase in the money supply results in an increase in output as well as in prices. In this case, the increase in the money supply is predominantly perceived as monetary expansion due to the uncertainty channel. However, when σ_{em}^2 is high, the direct welfare-reducing effect of the volatility of policy shock outweighs the indirect effect through the uncertainty channel.

Therefore, the effects of the money supply fluctuations are the results of both the direct and indirect effects of the volatility of monetary shocks. As in the complete information model, an increase in the volatility of policy shocks increases the volatility of output and prices. However, in this case, information frictions also change the qualitative response of the economy to fluctuations in the money supply. We can see from Fig. 3 that with low volatility of policy shocks, the central bank can trade off the decrease in prices with some loss in output. However, as the volatility of policy shocks increases, the direct welfare-reducing effect dominates and the fluctuations in output and prices are amplified.

As we study the response of the economy to money supply fluctuations under different values of $\sigma^2_{e^m}$, the intersections of the AD and AS curves trace out the set of feasible combinations of volatilities of output and prices as a function of $\sigma^2_{e^m}$. The right panel plots these feasible equilibria for different values of the policy response parameter ϕ along with the contours of the welfare loss function above. We can see from the graph that for a given value of ϕ the optimal policy implies a non-zero volatility of policy shocks that achieves the tangency point between the feasible volatility of the economy and welfare contours. I formalize this result in Theorem 2 below.

Theorem 2 (Optimal policy with public signal only)
Under incomplete information with public signal only, the optimal policy is characterized by the following policy parameters:

- *for a given ϕ, the optimal variance of the monetary policy shock $\sigma^2_{e^m}$ is equal to*

$$\sigma^{2*}_{e^m} = \phi\left(\Phi - \phi\right)\sigma^2_{e^u}$$

if $0 < \phi < \Phi$, where

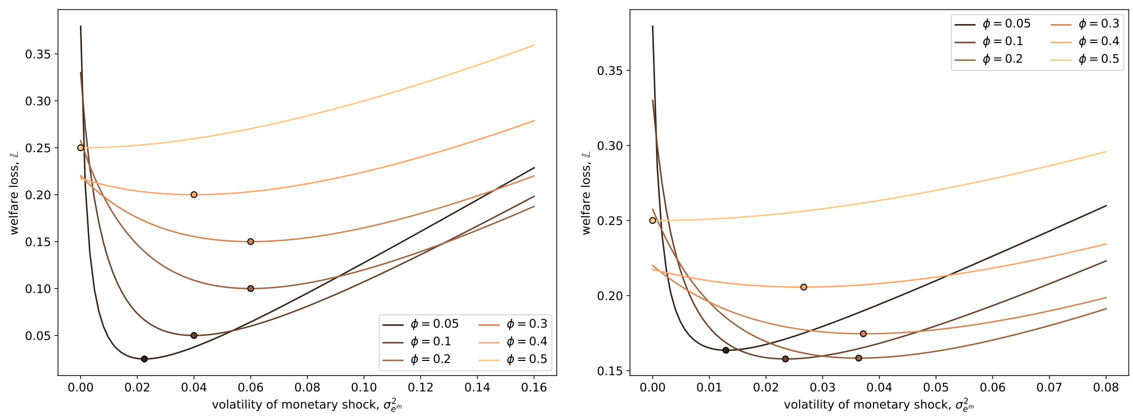
$$\Phi = \sqrt{\frac{(1-\theta)\left(\epsilon\theta + (1-\theta)\left(\sigma+\varphi\right)\right)}{\theta\left(\sigma+\varphi\right)\left(\epsilon\left(1-\theta\right)\left(\sigma+\varphi\right)+\theta\right)}}$$

and is equal to zero if $\phi \geq \Phi$.

- *nevertheless, the optimal policy response parameter ϕ^* is equal to zero.*

Theorem 2 derives the optimal policy parameters under incomplete information with only the public signal. The results provide some important insights into the optimal policy design with incomplete information. Firstly, for a given ϕ, provided that $0 < \phi < \Phi$, the optimal policy implies a positive volatility of monetary policy shock. Hence, monetary shocks have net welfare-improving effects in this setting. Therefore, the central bank may mitigate the adverse effects of inefficient shocks via the uncertainty channel.

Nevertheless, unlike the complete information model, the optimal policy response parameter ϕ^* is equal to zero. This result is due to the fact that since the public signal on the money supply is the only source of information about the markup shock, the central bank may shut down the signaling channel by setting $\phi = 0$ and not responding to the markup shock. This, in turn, means that firms do not observe and do not respond to markup shocks, which eliminates the need for the central bank to respond to that shock. Consequently, the optimal volatility of monetary shock is zero, since the uncertainty channel is no longer operational.



a) public signal only

b) both private and public signals

Figure 4: Welfare losses as a function of σ^2_{em} for different values of ϕ

I also plot the welfare losses as a function of σ^2_{em} for different values of ϕ in the left panel of Fig. 4. The graph illustrates the results of Theorem 2 and shows that welfare losses are decreasing in σ^2_{em} if $0 < \phi < \Phi$, while the optimal σ^2_{em} is zero if $\phi = 0$ or $\phi \geq \Phi$.

Next, I consider the full-scale model with both private and public signals and characterize the optimal policy in that setting. Unlike the setup with only a public signal, in this case, $\phi = 0$ cannot be optimal, since firms learn about the markup shock from the private signal even if the public signal is not informative.

Theorem 3 (Optimal policy with private and public signals)
Under incomplete information with private and public signals, the optimal policy is characterized by $0 < \phi^* < \Phi$ *and* $\sigma^{2*}_{em} > 0$ *or* $\phi^* \geq \Phi$ *and* $\sigma^{2*}_{em} = 0$ *depending on the parameters of the model.*

Corollary 3.1
For $\theta = \frac{\sigma+\varphi}{\sigma+\varphi+1}$ *and* $1 < \epsilon \leq 15$, *the optimal policy is characterized by* $0 < \phi^* < \frac{1}{\sigma+\varphi}$ *and* $\sigma^{2*}_{em} > 0$.

The richer setup with both private and public signals makes analytical derivations of the optimal policy parameters more challenging. Nevertheless, the results of Theorem 3 show that in this setting the optimal policy implies a strictly positive response parameter ϕ. Moreover, and most importantly, it shows that depending on the parameters of the model, the optimal policy may involve a non-zero volatility of monetary policy shocks. Furthermore, Corollary 3.1 provides an example of the parameter values for which the optimal policy implies $\phi^* > 0$ and $\sigma_{em}^{2*} > 0$.

The welfare losses with both private and public signals are plotted in the right panel of Fig. 4. The graph illustrates the results of Theorem 3 and shows that welfare losses are decreasing in σ_{em}^2 for $0 < \phi < \Phi$, and the optimal policy is given by $\sigma_{em}^{2*} > 0$, while the optimal σ_{em}^2 is zero for $\phi \geq \Phi$.

1.9 Policy implications

The results of the paper have important implications for the optimal policy design under incomplete information. The uncertainty channel of monetary policy allows the central bank to use the strategic uncertainty generated by the monetary policy shocks to reduce the adverse effects of inefficient shocks on the economy. The central bank may use the monetary policy shocks to dampen the effects of the inefficient shocks on the economy and reduce the welfare losses generated by these shocks.

This novel channel of monetary policy has several practical implications for policymaking. In particular, the optimal policy design implies that the central bank may deviate from the policy rule to reduce the effects of inefficient shocks on the economy. These deviations may take different forms, as long as they are orthogonal to the policy rule.

Firstly, apart from reacting to inflation and the output gap, the central bank may use the policy instrument (e.g. the money supply or the interest rate) to respond to other variables in the economy, such as exchange rate or asset prices.[9] Or the central bank may conduct discretionary policies instead of strictly following the policy rule.[10] Either way, the central bank may deviate from the policy rate implied by inflation and the output gap, while those deviations may have additional benefits for the economy through the uncertainty channel.

Secondly, the central bank may use non-linear policy rules to respond to inflation and the output gap. Then, if the public perceives the policy rule as linear, the residual deviations from the linear rule may take the form of deviations from the policy rule that are orthogonal

[9]Responses to asset prices or exchange rates are particularly relevant since these variables are typically unpredictable, thus the central bank's responses to them may take the form of similarly unpredictable policy shocks.

[10]For instance, Taylor (2012) defines discretionary policies as the ones that deviate from the course implied by the policy rule.

to the linear part of it. Similar to the previous case, these deviations provide additional benefits for social welfare through the uncertainty channel.

Lastly, the central bank may not observe the inflation and especially the output gap in real-time.[11] Thus, imperfect and noisy observations on these variables will imply subsequent deviations from the policy rule.[12] These deviations also fit under the general form of policy shocks that enable the operation of the uncertainty channel.

In all cases above, a necessary condition for the operation of the uncertainty channel is that the public perceives the policy rule as linear, which is supported by empirical evidence on the monetary policy rules. Additionally, from the private sector's point of view, the deviations from the policy rule should be orthogonal to the policy rule.

Another important consideration regarding the implementation of the proposed strategy is its *time consistency*. Although the presence of monetary policy shocks reduces the effects of markup shocks, monetary shocks generate economic fluctuations and directly reduce welfare. Under the proposed policy strategy, the central bank commits to deviations from the policy rule despite these costs.

At first, such a policy strategy may seem to trigger time inconsistency in the central bank's plans and actions. A policymaker will *ex-ante* find it optimal to commit to deviations from the policy rule to affect firm beliefs about the shocks, but *ex-post* will have no incentive to generate such disturbances. Nevertheless, I showed that the central bank achieves higher welfare by committing to such a policy strategy and delivering the promised deviations from the policy rule.

2 A dynamic model

In this section, I study the uncertainty channel of monetary policy in a dynamic New Keynesian model with incomplete information. The model economy is based on the dispersed information models developed by Nimark (2008) and Melosi (2016) and is populated by a representative household, a continuum of firms, and a monetary authority.

[11]See Orphanides (2003) for a discussion of the implications of noisy information for the use of policy rules in practice.

[12]For example, in Melosi (2016), the central bank has noisy observations of inflation and the output gap, which generate deviations from the policy rule.

2.1 Households

Households supply labor N_t to firms and consume C_t according to preferences given by

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

A representative household smooths its consumption over time by trading one-period risk-free securities. It maximizes its lifetime utility given by

$$\sum_{s=0}^{\infty} \beta^s E_t \left[U(C_{t+s}, N_{t+s}) \right]$$

subject to budget constraints given by

$$P_t C_t + Q_t B_{t+1} = W_t N_t + B_t + T_t$$

where P_t is the price level, Q_t is the price of bonds, B_t denotes bond holdings, W_t denotes the wage rate, and T_t are lump-sum transfers to the household, including profits from the firms.

First-order conditions for the utility maximization problem imply

$$\begin{aligned} Q_t &= \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] \\ N_t^{\varphi} &= C_t^{-\sigma} \frac{W_t}{P_t} \end{aligned} \tag{15}$$

2.2 Firms

Monopolistically competitive firms, indexed by j, produce differentiated goods using household labor. The production function of firms is given by

$$Y_{j,t} = A N_{j,t}^{1-\alpha}$$

where A is the technology, normalized to one, and α is the share of capital in the production function.

Firms set prices according to the Calvo (1983) framework, where each period they are allowed to reset their prices with probability $1-\theta$. Each firm, reoptimizing in period t, will choose a price $P_{j,t}^*$ that maximizes the expected value of real profits. Formally, they solve the problem

$$\max_{P^*_{j,t}} \quad \sum_{s=0}^{\infty} \theta^s E_{j,t} \left[\Lambda_{t,t+s} \left(P^*_{j,t} Y_{j,t+s|t} - \Psi_{t+s} \left(Y_{j,t+s|t} \right) \right) \right]$$

subject to the sequence of demand constraints

$$Y_{j,t+s|t} = \left(\frac{P^*_{j,t}}{P_{t+s}} \right)^{-\epsilon} Y_{t+s}$$

where $\Lambda_{t,t+s} = \beta^s \left(C_{t+s}/C_t \right)^{-\sigma} \left(P_t/P_{t+s} \right)$ is the stochastic discount factor of firms, $\Psi_{j,t+s}(\cdot)$ is the cost function of firm j, and $Y_{j,t+s|t}$ is the output of firm j, given that it optimized in period t. The expectation operator $E_{j,t}\left[\,\cdot\,\right]$ denotes the expectation of firm j and differs from the generic operator $E_t\left[\,\cdot\,\right]$ in terms of the information available to firms. I define the expectation operator of firm j as $E_{j,t}\left[\,\cdot\,\right] \equiv E\left[\,\cdot\,|\,\mathcal{I}_{j,t}\right]$.

The optimality condition implies

$$\sum_{s=0}^{\infty} \theta^s E_{j,t} \left[\Lambda_{t,t+s} Y_{j,t+s|t} \left(P^*_{j,t} - \mathcal{M}_t P_{t+s} MC_{j,t+s|t} \right) \right] = 0 \tag{16}$$

where $MC_{j,t+s|t} = \Psi'_{j,t+s} \left(Y_{j,t+s|t} \right) / P_{t+s}$ is the real marginal cost of firm j, and $\mathcal{M}_t$ is the desired markup of firms.

I also introduce a shock to desired mark-ups $\mathcal{M}_t$ in Eq. (16). In log-linear terms, desired mark-ups are assumed to follow an $AR(1)$ process of form

$$e^u_t = \rho_u e^u_{t-1} + \varepsilon^u_t \qquad\qquad \varepsilon^u_t \sim N(0, \sigma^2_u)$$

where $e^u_t = \log \mathcal{M}_t - \log \mathcal{M}$, and $\mathcal{M}$ is the steady-state level of mark-ups. I assume that labor income is subsidized by lump-sum taxes to restore the efficient steady-state level of output. With the subsidy in place, the steady-state level of mark-ups is given by $\mathcal{M} = 1$.[13]

Since markup shocks, and cost-push shocks in general, generate a wedge between natural and efficient levels of output, they are inefficient disturbances, as opposed to demand shocks or technology shocks.[14] Specifically, these shocks create a trade-off between inflation and output gap for the central bank, such that inflation stabilization does not simultaneously achieve output gap stabilization. Hence, for a benevolent central bank, these shocks are

[13] See Galí (2015) for a discussion of the distortion of the efficient steady state due to monopolistic competition.

[14] See Woodford (2003b) and Galí (2015) for a discussion of the role of inefficient shocks in New Keynesian models.

undesirable and should be minimized. This feature of the model plays a crucial role in the analysis of optimal monetary policy in the presence of information frictions.

2.3 Central bank

Unlike firms, the central bank is assumed to have complete information. It sets the nominal interest rate according to a Taylor rule responding to inflation and output.

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \exp\left(e_t^m\right) \tag{17}$$

where R, Π, and Y are the steady-state levels of the interest rate, inflation, output, and e_t^m is a monetary policy shock.

The monetary shock e_t^m follows an $AR(1)$ process

$$e_t^m = \rho_m e_{t-1}^m + \varepsilon_t^m \qquad\qquad \varepsilon_t^m \sim N(0, \sigma_m^2)$$

In this formulation, the central bank responds to the output gap and inflation with coefficients ϕ_y and ϕ_π, respectively, and deviates from the policy rule by the monetary shock e_t^m. In Section 4, I discuss the role of this shock and show that the optimal policy is characterized by a strictly positive volatility of the policy shock.

2.4 Market-clearing

The labor market clears at wage rate W_t such that

$$N_t = \int_j N_{j,t}\, dj \tag{18}$$

Product markets clear with

$$Y_{j,t} = C_{j,t} \tag{19}$$

Finally, the bond market clears with zero net bond holdings

$$B_t = 0 \tag{20}$$

2.5 Equilibrium

To introduce incomplete information into the model, we assume that each period t consists of two stages. Throughout both stages, households and the central bank have complete information, however, each firm's information set consists of a history of signals that it receives at the beginning of each time period.

Firm price-setting behavior under incomplete information is modeled by a two-stage decision-making framework, similar to Nimark (2008). In this setup, we assume that firms choose their optimal prices based on the limited information available to them at the beginning of each period. The sequence of events and the decision-making process is illustrated in Fig. 5.

In the first stage, each firm j receives signals $S_{j,t}$ on the state of the economy. Then, each firm sets its price $P_{j,t}$ based on its information set $\mathcal{I}_{j,t}$, without observing its marginal costs or demand for its product. The information set is presented in more detail in the next section.

In the second stage, each firm j hires labor $N_{j,t}$ and meets the demand for its product $Y_{j,t}$. At this stage, firms cannot re-optimize after observing demand or wages but simply supply the demand at the price $P_{j,t}$ set in the first stage.

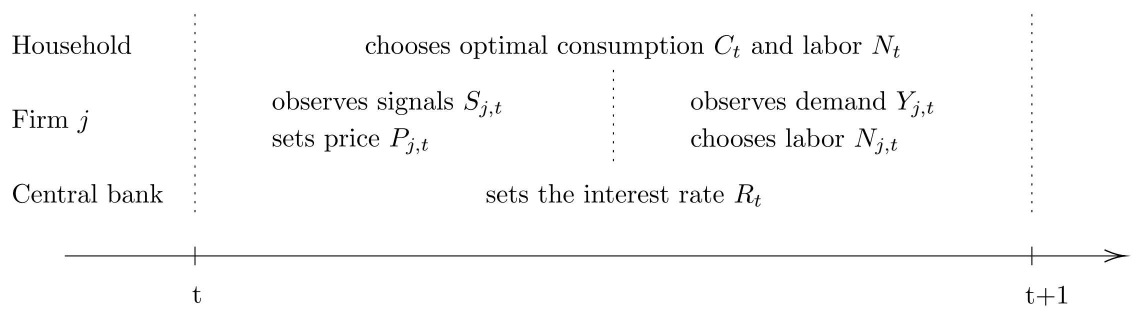


Figure 5: The sequence of events in each time period

I define the competitive equilibrium of the economy as similar to that of a standard New Keynesian economy, except for the information available to each economic agent and the decision-making process described above.

Definition 2 (Equilibrium)
For sequences of exogenous disturbances e_t^u and e_t^m, competitive equilibrium is defined as the sequences of $P_{j,t}$, $Y_{j,t}$, $N_{j,t}$, P_t, Y_t, N_t, C_t, B_t, Q_t, and R_t, such that

- *the representative household chooses consumption C_t and labor N_t according to the optimality conditions in Eq. (15)*

- *each firm j sets its price $P_{j,t}$ according to Eq. (16) given its information set $\mathcal{I}_{j,t}$, then hires labor $N_{j,t}$ and meets demand $Y_{j,t}$ at the price $P_{j,t}$*

- *the central bank sets the nominal interest rate R_t according to the rule in Eq. (17)*

- *markets clear according to conditions Eqs. (18), (19) and (20)*

2.6 Information Structure

I assume that households and the central bank have complete information but firms do not. Each firm has to infer the state of the economy based on the information available to them.

In the first stage of each period t, given the information as of the second stage of the previous period $\mathcal{I}^2_{j,t-1}$, firm j observes a vector of signals on the current state of the economy. I assume that firms observe noisy signals about the markup shock and the interest rate

$$S^{e^u_t}_{j,t} = e^u_t + \zeta^{e^u_t}_{j,t} \qquad \zeta^{e^u_t}_{j,t} \sim N\left(0, \sigma^2_{\zeta_{e^u_t}}\right)$$
$$S^{i_t}_{j,t} = i_t + \zeta^{i_t}_{j,t} \qquad \zeta^{i_t}_{j,t} \sim N\left(0, \sigma^2_{\zeta_{i_t}}\right)$$

The signals observed in the first stage, along with the information as of the second stage of the previous period, form the firm's new information set $\mathcal{I}^1_{j,t}$, such that

$$\mathcal{I}^1_{j,t} = \left\{S^{e^u_t}_{j,t}, S^{i_t}_{j,t}\right\} \cup \mathcal{I}^2_{j,t-1}$$

In the second stage, firm j hires labor $N_{j,t}$ at wage rate W_t and accommodates demand $Y_{j,t}$ at price $P_{j,t}$. In log-linear forms, the wage rate and the demand are given by

$$w_t = (\sigma + \varphi/(1-\alpha))\, y_t + p_t \qquad y_{j,t} = -\epsilon\,(p_{j,t} - p_t) + y_t$$

Given firms' knowledge about its price $p_{j,t}$ and beliefs about p_{t-1}, the observed wage rate and demand w_t and $y_{j,t}$ are effective signals about y_t and π_t, such that

$$w_t = p_{t-1} + (\sigma + \varphi/(1-\alpha))\, y_t + \pi_t \qquad y_{j,t} = -\epsilon\,(p_{j,t} - p_{t-1}) + y_t + \epsilon\pi_t$$

Hence, observing w_t and $y_{j,t}$ is almost equivalent to observing y_t and π_t. Therefore, I assume that in the second stage of each period, firms also receive noisy signals about current output, inflation, and interest rate, given by

$$
\begin{aligned}
S_{j,t}^{y_t} &= y_t + \zeta_{j,t}^{y_t} \qquad & \zeta_{j,t}^{y_t} \sim N\left(0, \sigma_{\zeta_{y_t}}^2\right) \\
S_{j,t}^{\pi_t} &= \pi_t + \zeta_{j,t}^{\pi_t} \qquad & \zeta_{j,t}^{\pi_t} \sim N\left(0, \sigma_{\zeta_{\pi_t}}^2\right) \\
S_{j,t}^{i_t} &= i_t + \zeta_{j,t}^{i_t} \qquad & \zeta_{j,t}^{i_t} \sim N\left(0, \sigma_{\zeta_{i_t}}^2\right)
\end{aligned}
$$

such that

$$
\mathcal{I}_{j,t}^2 = \{S_{j,t}^{y_t}, S_{j,t}^{\pi_t}, S_{j,t}^{i_t}\} \cup \mathcal{I}_{j,t}^1
$$

Then, the first-stage information set of each firm j can be written recursively as

$$
\mathcal{I}_{j,t} = \left\{S_{j,t}^{e_t^u}, S_{j,t}^{i_t}, S_{j,t}^{y_{t-1}}, S_{j,t}^{\pi_{t-1}}, S_{j,t}^{i_{t-1}}\right\} \cup \mathcal{I}_{j,t-1}
$$

where $\mathcal{I}_{j,t} \equiv \mathcal{I}_{j,t}^1$ denotes the information set of firm j at the beginning of period t and is comprised of the current period markup shock signal and the interest rate signal, along with the signals about prior output, inflation, interest rate, and the information set of the previous period $\mathcal{I}_{j,t-1}$.

2.7 Log-linearization and aggregation

Log-linear approximation of the optimality conditions of households in Eq. (15) around the perfect-foresight steady state yields

$$
\begin{aligned}
c_t &= E_t[c_{t+1}] - \frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}]\right) \\
\sigma c_t + \varphi n_t &= w_t - p_t
\end{aligned}
$$

On the other hand, log-linearization of the first-order condition of firms in Eq. (16) yields

$$
p_{j,t}^* = (1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s E_{j,t}\left(mc_{j,t+s|t} + p_{t+s} + e_{t+s}^u\right)
$$

where $e_{t+s}^u = \log \mathcal{M}_{t+s} - \log \mathcal{M}$ is the log deviation of desired markup and $mc_{j,t+s|t} = \log MC_{j,t+s|t} - \log MC$ is the log deviation of real marginal cost of firm j which optimized at time t from their steady state values at time $t+s$.

Aggregating the first-order conditions of firms and using the definition of inflation yields the following Phillips curve equation

$$\pi_t = \beta\theta \sum_{k=0}^{\infty} (1-\theta)^k \,\overline{E}_t^{(k+1)} \left[\pi_{t+1}\right] + (1-\theta)\,(1-\beta\theta)\,\Theta \sum_{k=0}^{\infty} (1-\theta)^k \,\overline{E}_t^{(k+1)} \left[mc_t\right] +$$

$$+ (1-\theta)\,(1-\beta\theta)\,\Theta \sum_{k=0}^{\infty} (1-\theta)^k \,\overline{E}_t^{(k+1)} \left[e_t^u\right]$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

In log-linear terms, the central bank sets the interest rate according to

$$i_t = \phi_\pi \pi_t + \phi_y y_t + e_t^m \tag{21}$$

In summary, the model economy is described by the following set of equations

$$y_t = E_t[y_{t+1}] - \frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}]\right)$$

$$\pi_t = \beta\theta \sum_{k=0}^{\infty} (1-\theta)^k \,\overline{E}_t^{(k+1)} \left[\pi_{t+1}\right] + (1-\theta)\,(1-\beta\theta)\,\Theta\Lambda \sum_{k=0}^{\infty} (1-\theta)^k \,\overline{E}_t^{(k+1)} \left[y_t\right] +$$

$$+ (1-\theta)\,(1-\beta\theta)\,\Theta \sum_{k=0}^{\infty} (1-\theta)^k \,\overline{E}_t^{(k+1)} \left[e_t^u\right]$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t + e_t^m$$

where $\Lambda = \sigma + \frac{\varphi+\alpha}{1-\alpha}$.[15]

The set of equations above characterizes the aggregate dynamics of the economy. It is important to note that the model economy nests a standard New Keynesian model with complete information as a special case, where $\overline{E}_t^{(k)}\left[x_t\right] = x_t$ for $k \geq 0$.

2.8 Solution

I calibrate the model parameters using standard values in the New Keynesian literature. For the calibration of the model parameters presented in Table 1, I set the parameter values to those used in Galí (2015).

[15] See the Online Appendix for the derivation of the model equations.

Parameter		Value
Discount factor	β	0.99
Intertemporal elasticity of substitution	σ	1
Inverse Frisch elasticity of labor supply	φ	1
Capital share	α	1/3
Price stickiness	θ	2/3
Elasticity of substitution between goods	ϵ	6
Central bank reaction to inflation	ϕ_π	1.5
Central bank reaction to output gap	ϕ_y	0.5/4

Table 1: Parameter values of deep model parameters

The calibration of parameters describing the shock process and signals observed by firms is presented in Table 2. The persistence parameter of the markup shock is set to 0.7, which is common in similar studies, while the standard deviation of innovations to the markup shock is set to 1.5. I set the standard deviation of the noise in the markup shock signal to 1.5 and the standard deviation of the noise in all signals about lagged aggregate variables to 0.5.

Parameter		Value
Persistence of markup shock	ρ_u	0.7
Standard deviation of markup shock	σ_u	1.5
Standard deviation of noise in markup shock signal	$\sigma_{\zeta_{e_t^u}}$	1.5
Standard deviation of noise in lagged output signal	$\sigma_{\zeta_{y_{t-1}}}$	0.5
Standard deviation of noise in lagged inflation signal	$\sigma_{\zeta_{\pi_{t-1}}}$	0.5
Standard deviation of noise in lagged interest rate signal	$\sigma_{\zeta_{i_{t-1}}}$	0.5

Table 2: Parameter values governing shock processes and signals

The model is solved by the method of undetermined coefficients using an iterated solution algorithm similar to Nimark (2008).

Let X_t be a vector of shocks driving the economy. In the benchmark model with only markup shocks, we have $X_t = [e_t^u, e_t^m]'$. Define a vector collecting the hierarchy of firms' beliefs, given by

$$X_t^{(0:\infty)} = \left[X_t^{(0)'} \quad X_t^{(1)'} \quad X_t^{(2)'} \quad \ldots \right]'$$

where

$$X_t^{(0)} = X_t \qquad X_t^{(k)} = \overline{E}_t \left[X_t^{(k-1)} \right] \quad \text{for} \quad k = 1, 2, \ldots$$

The vector $X_t^{(0:\infty)}$ comprising beliefs in the economy represents the state vector that will determine the equilibrium in the model. I conjecture and later verify in the Online Appendix that the state vector $X_t^{(0:\infty)}$ evolves according to

$$X_t^{(0:\infty)} = M X_{t-1}^{(0:\infty)} + N v_t$$

where M and N are coefficient matrices to be found and $v_t = [\varepsilon_t^u, \varepsilon_t^m]'$ is a vector of innovations to the exogenous processes.

Also, let $z_t = [y_t, \pi_t, i_t]'$ be a vector of output, inflation, and interest rate. I conjecture and then verify that the model solution takes the form of

$$z_t = A X_t^{(0:\infty)}$$

where A is a coefficient matrix that maps the entire belief hierarchy of households, firms, and the central bank to the equilibrium values of output, inflation, and interest rate.

The coefficient matrices A, M, N are found by iterating the equations above using the procedure described in the Online Appendix.

3 The signaling and uncertainty channels

3.1 Interest rate signaling with markup shocks

In this section, I study the effects of markup shocks on the model economy, benchmarking it with a standard New Keynesian model with complete information. First, I assume that firms receive noisy signals about past output, inflation, and interest rate as well as a signal on markups, but not the contemporaneous interest rate. I begin by comparing the effects of markup shocks under complete and dispersed information. Fig. 6 presents the response of the economy to a one standard deviation shock to mark-ups under each setting.[16]

[16]For comparison, in the Online Appendix, I present the impulse response functions of the model with one-period uncertainty.

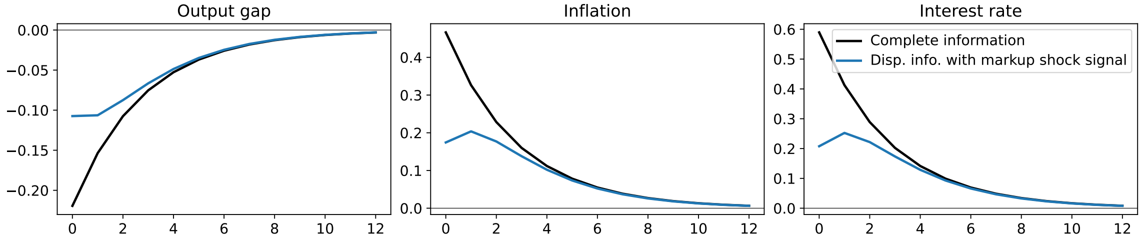


Figure 6: Impulse response functions of a 1 s.d. markup shock

As can be seen from the figure, the contemporaneous impact of the shock is smaller under dispersed information. This effect naturally arises because with incomplete information firms respond to the shock less, due to the presence of the noise in firms' signals and underestimation of the shock. Consequently, firms increase their prices by less than they do with complete information. As firms learn about the true value of the shock, they adjust their prices accordingly and the economy converges to the complete information benchmark over time.

Let's now assume the interest rate set by the central bank can be observed before setting prices. Observing the interest rate provides additional information about the economy. The central bank can use this channel to transmit information about the shock via its policy rate. This phenomenon, known in the literature as *interest rate signaling*, has been studied by Melosi (2016), who estimates a similar model with dispersed information for the US economy.

To study the effects of interest rate signaling in my model, I also plot the response of the economy to a markup shock with and without the interest rate signal $S_{j,t}^{i_t}$ in Fig. 7.

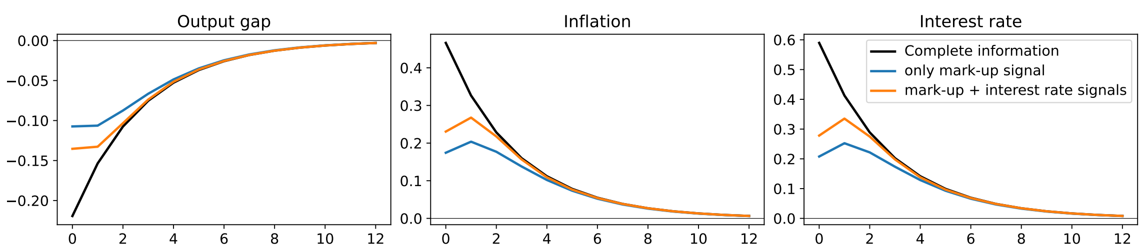


Figure 7: Impulse response functions of a 1 s.d. markup shock

Fig. 7 shows that the interest rate signaling channel of monetary policy increases the information available to firms and results in larger price increases in response to a markup shock. As a result, the contemporaneous impact of the shock is larger relative to the economy with only signals on the markup shock. Additionally, with both signals firms learn about the true value of the shock at a faster rate and the economy converges to the complete information benchmark more quickly.

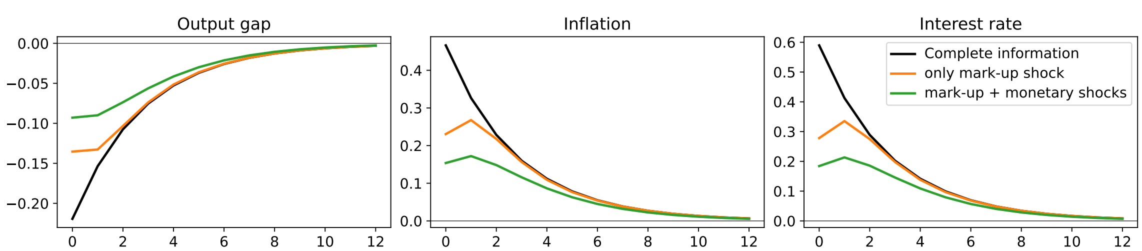
3.2 Monetary shocks and the uncertainty channel

In Section 1, I showed that the response of the economy to the markup shock depends on the volatility of the monetary shock. Hence, in this section, I study the same phenomenon in the dynamic model with monetary shocks in the interest rate rule.

Recall that the policy rule of the central bank is given by

$$\begin{aligned} i_t &= \phi_\pi \pi_t + \phi_y y_t + e_t^m \\ e_t^m &= \rho_m e_{t-1}^m + \varepsilon_t^m \qquad\qquad \varepsilon_t^m \sim N\left(0, \sigma_m^2\right) \end{aligned} \tag{22}$$

where ρ_m is the persistence of monetary shocks and σ_m^2 is the variance of monetary shocks. Under a purely rule-based policy, we have $\sigma_m^2 = 0$, while under the proposed policy framework, $\sigma_m^2 > 0$. I present the effects of markup shocks under the two policies in Fig. 8. With monetary shocks, the information extraction problem faced by firms requires them to infer both the size and type of shock, increasing their net uncertainty and moderating their pricing.

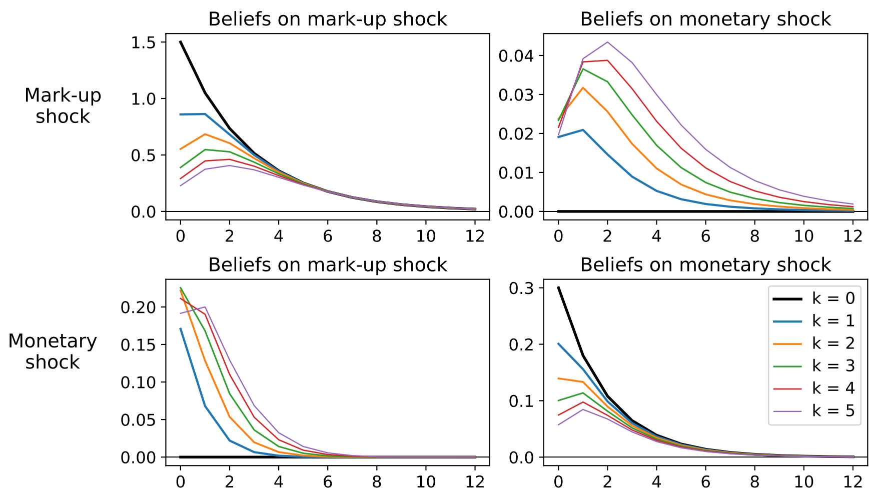


Note: Effects of a 1 s.d. markup shock in two different economies - an economy with only markup shocks, i.e. $\sigma_m^2 = 0$ (green), and in an economy with markup and monetary shocks, i.e. $\sigma_m^2 > 0$ (orange).

Figure 8: Impulse response functions of a 1 s.d. markup shock

If the central bank follows a policy strategy that involves monetary shocks, then it can potentially affect firms' beliefs about the markup shock by systematically deviating from the policy rule. If changes in the policy rate are not only driven by the central bank's reaction to inflation and output gap but also by monetary shocks, then firms cannot infer the type of shock from the policy rate. Therefore, the uncertainty about the type of the shock increases and makes firms' beliefs about the markup shock more dispersed due to the *uncertainty channel* of monetary policy.

Under the proposed policy strategy, firms have to infer the type of the shock as well as its size, and as a result, the response of the economy becomes a mix of the responses to markup and monetary shocks under complete information. To see how the uncertainty about the shocks affects firms' beliefs, consider the impulse response functions of firms' beliefs about both shocks in Fig. 9.



Note: Effects of 1 s.d. markup and monetary shocks on the k-th order beliefs of firms.

Figure 9: Impulse response functions of firms' beliefs after markup and monetary shocks

Fig. 9 plots the mean beliefs of firms over both markup and monetary shocks after the separate realizations of each type of shock. Firms cannot distinguish shocks perfectly and so each induces mixed beliefs with corresponding weights on both possibilities. As firms continue receiving signals, uncertainty is resolved and beliefs converge to the true shock. However, because of the mixed beliefs, the economic responses to both shocks are altered.

The effects of monetary shocks on an economy can be summarized as follows. First, via the uncertainty channel, the presence of monetary shocks reduces the effects of markup shocks. This is an indirect effect of monetary shocks that affect firm beliefs about the output gap. Hence, the uncertainty channel improves the inflation-output gap trade-off and is thus welfare-improving. Second, the monetary shocks themselves generate economic fluctuations and directly reduce welfare. Therefore, optimal monetary policy depends on the relative costs and benefits of the shocks. In Section 4, I evaluate the welfare losses as a function of monetary policy to identify the optimal policy approach.

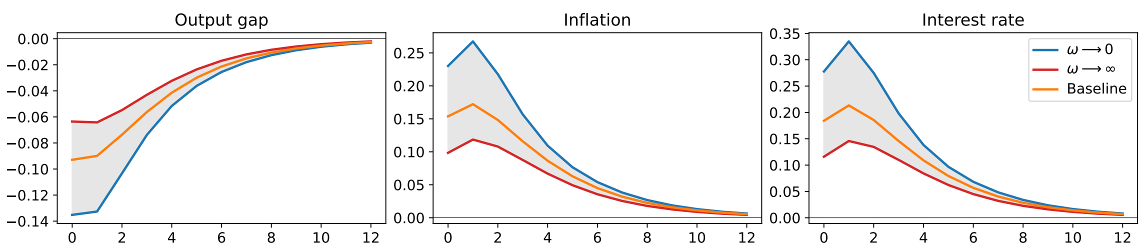
3.3 High versus low monetary policy uncertainty

In Section 1, I showed that in the simple model, the effects of markup and monetary shocks are significantly different under different values of σ_m^2. Hence, in this section, I study the response of the model economy to both shocks with different volatilities of the policy shock.

To study the differences in the propagation of shocks, I define a measure of monetary policy uncertainty as the standard deviation of monetary shocks relative to the standard deviation of markup shocks, as $\omega \equiv \sigma_m/\sigma_u$. Next, I consider two monetary regimes: high and low. The high uncertainty regime is defined as $\omega \to \infty$, i.e., the economy is mostly driven by monetary shocks. Similarly, the low uncertainty regime is defined as $\omega \to 0$, i.e., the economy is mostly driven by markup shocks.

Under the high monetary policy uncertainty regime, given the signals received, firms will tend to attribute most signal variability to monetary shocks. Regardless of the type of shock, both markup and monetary shocks will be perceived as monetary shocks. Similarly, with low monetary policy uncertainty, both shocks will be perceived as markup shocks and the responses will change accordingly.

To study the effects of monetary policy uncertainty on the response of the economy to either shock, I plot the impulse responses of the output gap and inflation to a markup shock under high and low monetary policy uncertainty regimes in Fig. 10.

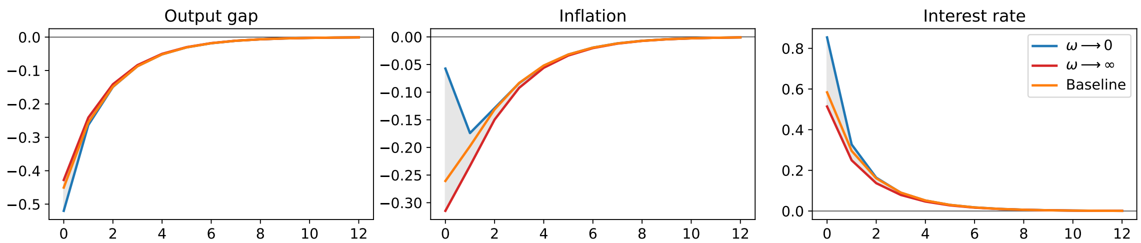


Note: The response of the economy to a 1 s.d. markup shock under high (red) and low (blue) monetary policy uncertainty regimes, versus the baseline model.

Figure 10: Impulse response functions of a 1 s.d. markup shock under low/high monetary policy uncertainty regimes

As can be seen from the figure, the response to a markup shock is qualitatively similar but quantitatively different depending on monetary uncertainty. As the volatility of monetary shocks increases, the response to a markup shock weakens. This is due to the uncertainty channel, because of which a true markup shock is perceived as a monetary shock. As a result, the response is a mix (weighted average) of the responses to markup and monetary shocks under complete information. This result is in line with the implications of Proposition 2 for the simple model.

Next, I plot the impulse responses of the output gap and inflation to a monetary shock under the high and low monetary policy uncertainty regimes in Fig. 11.



Note: The response of the economy to a 1 s.d. monetary policy shock under high (red) and low (blue) monetary policy uncertainty regimes, versus the baseline model.

Figure 11: Impulse response functions of a 1 s.d. monetary shock under low/high monetary policy uncertainty regimes

Similar to the effects of a markup shock, a monetary shock generates a different response under high and low monetary policy uncertainty regimes. In the case of low policy uncertainty, a true monetary shock is perceived as a markup shock. As a result, the response of the economy is biased towards the response to a true markup shock under complete information, shown in Fig. 6. As with markup shocks, the differences in the responses to monetary shock with different levels of policy uncertainty are consistent with ones given in Proposition 2 for the simple model.

The magnitude of the differences, in particular for the output gap, is small since both markup and monetary shocks generate a negative output gap, hence the observed effects under incomplete information are not very different from complete information. On the other hand, the differences in effects on inflation are larger, since under complete information inflation increases in response to a markup shock and decreases in response to a monetary shock. Nevertheless, the magnitude of these differences depends on the parameters of the model and may be larger under a different parameterization.

Based on the results obtained above, I formulate the following proposition, to be tested empirically in Section 5.

Proposition 3 (Effects of monetary policy uncertainty)
Consider two economies, denoted by L and H, such that $\omega_L < \omega_H$, i.e., in economy L monetary shocks are less dominant than in economy H.

Then

- *in case of a positive markup shock*

$$
\begin{aligned}
IRF_L(y_{t+h}) \mid e_t^u &< IRF_H(y_{t+h}) \mid e_t^u \\
IRF_L(\pi_{t+h}) \mid e_t^u &> IRF_H(\pi_{t+h}) \mid e_t^u
\end{aligned}
$$

- *in case of a positive monetary shock*

$$\begin{aligned} IRF_L(y_{t+h}) \mid e_t^m &< IRF_H(y_{t+h}) \mid e_t^m \\ IRF_L(\pi_{t+h}) \mid e_t^m &> IRF_H(\pi_{t+h}) \mid e_t^m \end{aligned}$$

for all $h \geq 0$.

The proposition formalizes the results illustrated in Figs. 10 and 11. It states that under the high monetary policy uncertainty regime, a positive markup shock results in a smaller decline in output and a smaller increase in inflation. Similarly, a positive monetary shock results in a smaller decline in output and a larger decline in inflation under the high monetary policy uncertainty regime.

4 Optimal policy design

The welfare-reducing effects of interest rate signaling are an important feature in the model that exists regardless of the central bank's information-disclosure policy. Although the monetary authority may find it beneficial to provide or suppress its private information about macroeconomic shocks, it cannot directly control the information transmitted by its policy rate. Therefore, optimal monetary policy should internalize the informational effects of monetary actions. In this section, I study optimal monetary policy design in the model implemented by a benevolent central bank that maximizes household welfare.

For this purpose, I use the second-order approximation of the household welfare around the perfect-foresight steady state, given by

$$\mathbb{W} = -\frac{1}{2} E \sum_{t=0}^{\infty} \beta^t \left[\Lambda y_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right] \tag{23}$$

where $\Lambda = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ and $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$.[17] [18]

Then, the average welfare loss per period will be given by

$$\mathbb{L} = \frac{1}{2} \left[\Lambda Var\left(y_t\right) + \frac{\epsilon}{\lambda} Var\left(\pi_t^2\right) \right] \tag{24}$$

As discussed in the previous section, markup shocks generate larger fluctuations under

[17] For the derivation of the household welfare function in Eq. (23), see Galí (2015).

[18] Similar to the simple model, the dynamic model also features additional price dispersion due to the idiosyncratic noise in the signals. However, in this section, I do not consider those effects since the numerical solution does not permit evaluation of this additional price dispersion.

complete information. As a result, the informativeness of the interest rate signal affects the response of the economy to the shocks and thus household welfare. To assess the welfare effects of interest rate signaling, I compare the welfare losses under complete information and dispersed information with interest rate signaling.

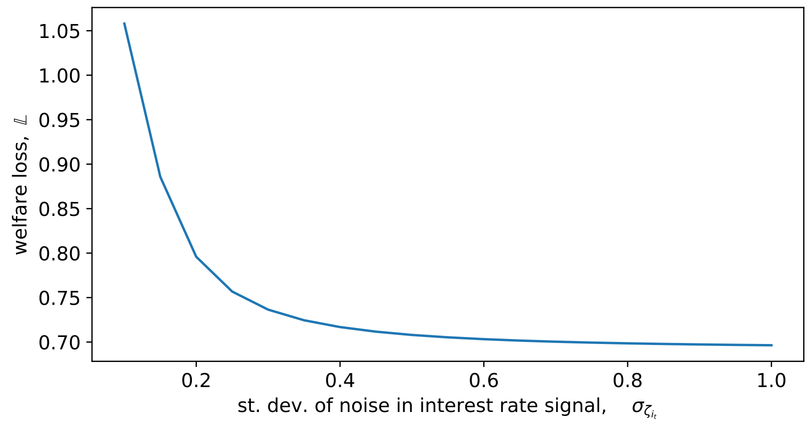


Figure 12: Welfare losses as a function of the standard deviation of noise in interest rate signal

Fig. 12 presents the welfare losses as a function of the standard deviation of noise in the interest rate signal. The welfare losses under complete information are significantly larger than under dispersed information with interest rate signaling. This result is consistent with the findings of the previous section, where I showed that the interest rate signaling channel makes the economy more responsive to shocks. Therefore, the central bank should take into account the informational effects of its actions while designing optimal monetary policy.

To further study the interaction between the interest rate signaling channel and the conventional interest rate channel, I consider a simple monetary policy rule of the form

$$i_t = \phi_\pi \pi_t + e_t^m \tag{25}$$

where for the sake of simplicity I assume $\phi_y = 0$, i.e. the interest rate does not react to output.

4.1 The optimal volatility of monetary shocks

The optimality of a policy strategy featuring monetary shocks depends on the relative costs and benefits of those disturbances. Hence, to analyze the potential welfare-improving effects of monetary policy shocks, I evaluate the welfare under different levels of volatility of monetary shocks. In Fig. 13, I plot the welfare losses as a function of the standard deviation of the monetary policy shocks using the complete and dispersed information models. In a standard New Keynesian model with complete information, the uncertainty channel of monetary policy is absent and therefore the policy shocks have strictly negative effects. However, with dispersed information, the benefits of monetary shocks outweigh the implied welfare costs and policy shocks have net positive effects.

The right panel of Fig. 13 demonstrates the welfare gains from a moderate volatility monetary shock in the dispersed information model. The welfare losses in the absence of monetary disturbances (i.e. when $\sigma_m^2 = 0$) are higher than with monetary shocks characterized by a standard deviation ranging from 0 to 0.4. For reference, the quarterly standard deviation of Romer and Romer (2004) shocks for the period 1969-1996 is 0.2, indicating the empirical plausibility of such gains.

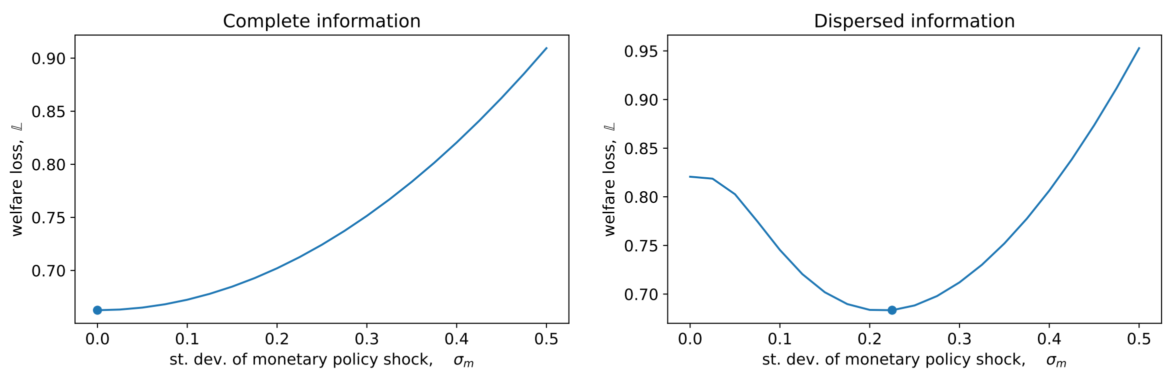


Figure 13: Welfare losses as a function of the standard deviation of the monetary shock

For the exposition, Fig. 13 plots the welfare losses setting the autocorrelation parameter of the monetary shocks to zero, i.e. $\rho_m = 0$. However, deviations from the policy rule are typically found to be persistent with a positive correlation.[19] In the left panel of Fig. 14, I plot the welfare losses as a function of the standard deviation of the monetary policy shock for different values of the autocorrelation parameter ρ_m. The optimal volatility of monetary shocks is decreasing in ρ_m as persistent monetary shocks create more undesirable fluctuations and are thus more costly.

On the other hand, since the uncertainty and interest rate channels of monetary policy operate through the policy rate, the benefits from uncertainty depend on the interest rate channel. In the right panel of Fig. 14, I plot the welfare losses as a function of the standard deviation of the monetary policy shock for different values of the policy response parameter ϕ_π.

[19] Orphanides (2004) finds evidence of serial correlation in monetary shocks, while Smets and Wouters (2007) estimates the coefficient of autocorrelation at 0.15 in the US. Galí (2015) calibrates this parameter to 0.5 in a simple New Keynesian model.

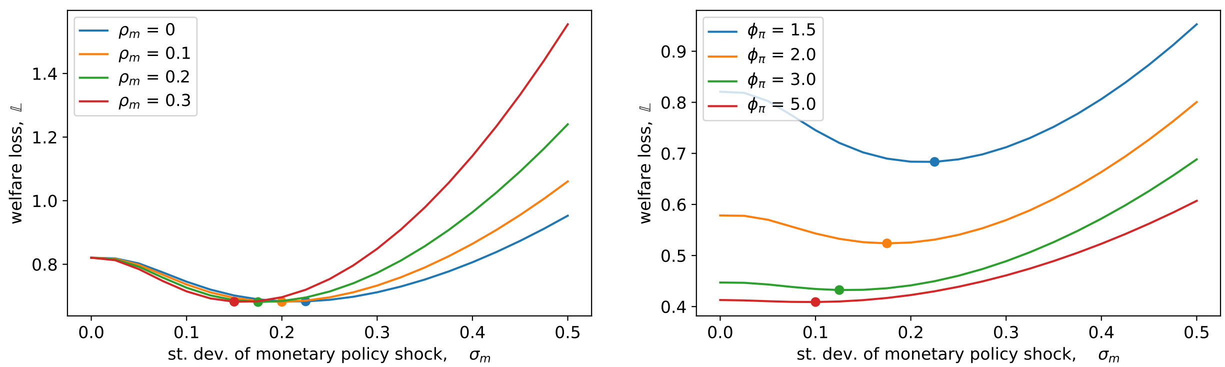


Figure 14: Welfare losses as a function of the standard deviation of monetary shock for different values of ρ_m and ϕ_π

Fig. 14 shows that the optimal volatility of monetary shocks is decreasing in the policy response parameter ϕ_π. This is because the interest rate channel of monetary policy is more effective in stabilizing the economy when the central bank responds more aggressively to inflation. Hence, the benefits of uncertainty are lower when the response of the central bank to inflation is stronger. Therefore, if a central bank can commit to an aggressive response to inflation, it should also minimize the volatility of monetary shocks. However, numerous empirical studies estimate the policy response parameter to be moderate, with values typically ranging from around 1 to 3, depending on the specification of the rule and the sample period.[20] In this case, the central bank can benefit from the uncertainty channel of monetary policy by increasing the volatility of monetary shocks.

As discussed previously, the welfare losses in the dispersed information model depend on the precision of the signals received by firms. Hence, the optimal volatility of monetary shocks depends on the noisiness of both firms' private signals about the markup shock and the public interest rate signal. In Fig. 15, I plot the welfare losses as a function of the standard deviation of the monetary policy shock for different values of the standard deviation of noise in both signals.

[20]For instance, Clarida et al. (2000) estimates this parameter at 0.83 for the pre-Volcker period and 2.15 for the Volcker-Greenspan period in the US.

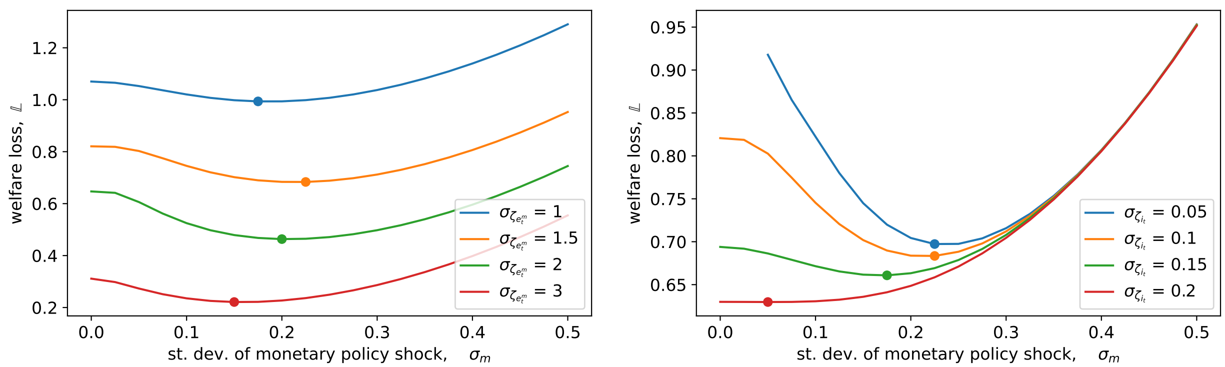


Figure 15: Welfare losses as a function of the standard deviation of monetary shock for different values of $\sigma_{\zeta_{e^u_t}}$ and $\sigma_{\zeta_{i_t}}$

As can be seen from the figure, the optimal volatility of monetary shocks is low when the private signal is either very noisy or very precise. If the private signal is very precise, firms know the markup shock regardless of what the central bank does and the uncertainty channel is less effective. On the other hand, if the private signal is very noisy, firms do not learn about the shock and do not react to it, so the welfare costs of markup shocks are low and the uncertainty channel is less effective.

The strength of the interest rate signaling channel is moderated by the noise in the signal, which may reflect firm inattention. The noisier the signal, the less attention firms pay to it. As can be seen from Fig. 15, the optimal volatility of monetary shocks is decreasing in the standard deviation of noise in the interest rate signal. This is because the noisiness of the signal (or, equivalently, firm inattention to the signal) undermines the signaling channel of monetary policy. Hence, the noisier the signal is, the lower the costs of monetary signaling, and thus the benefits of the uncertainty channel will be.

5 Empirical evidence

In this section, I provide empirical evidence in support of the model results and particularly Proposition 3.

The effects of monetary policy shocks on the economy summarized in Proposition 3 may be formulated as

$$\begin{aligned} y_{t+h} &= \gamma^{y,m}_{L,h}\, \varepsilon^m_t \cdot \mathbf{1}_{\omega<\overline{\omega}} + \gamma^{y,m}_{H,h}\, \varepsilon^m_t \cdot \mathbf{1}_{\omega>\overline{\omega}} + \boldsymbol{\beta}^{y,m}\, \boldsymbol{x}_t + \eta^{y,m}_{t+h} \\ \pi_{t+h} &= \gamma^{\pi,m}_{L,h}\, \varepsilon^m_t \cdot \mathbf{1}_{\omega<\overline{\omega}} + \gamma^{\pi,m}_{H,h}\, \varepsilon^m_t \cdot \mathbf{1}_{\omega>\overline{\omega}} + \boldsymbol{\beta}^{\pi,m}\, \boldsymbol{x}_t + \eta^{\pi,m}_{t+h} \end{aligned}$$

where ε_t^m is the monetary policy shock and $\overline{\omega}$ is a threshold for relative volatility of monetary policy and markup shocks. In the specifications above, the model predictions imply $\gamma_{L,h}^{y,m} < \gamma_{H,h}^{y,m}$ and $\gamma_{L,h}^{\pi,m} > \gamma_{H,h}^{\pi,m}$ for all $h \geq 0$.

I test these predictions by estimating the effects of monetary policy shocks during periods of high and low monetary policy uncertainty in the United States.

5.1 Data

To estimate the effects of monetary policy shocks, I use quarterly data on the US economy from 1970 to 2018. The data consists of the following variables: real GDP, industrial production, GDP deflator, consumer price index, and producer price index obtained from the FRED database of the Federal Reserve Bank of St. Louis.

I follow Romer and Romer (2004) to obtain a measure of monetary policy shocks. The resulting measures are particularly suitable for this exercise since they are constructed as residuals from an estimated policy rule similar to the monetary rule in the model. Unlike other popular measures of monetary shocks obtained by high-frequency identification methods, these series represent deviations of the policy rate from the estimated policy rule, rather than monetary policy surprises.

To construct policy shocks, I take the series obtained by Romer and Romer (2004) and extend it until 2018 using the Greenbook projections for the corresponding period. Since the sample contains a long period where the Fed funds rate was near zero, I use the Krippner (2016) shadow rate for the zero lower bound period.

To obtain a measure of monetary policy uncertainty, consider a GARCH(1,1) model of the form

$$\varepsilon_t = \mu + \epsilon_t \qquad\qquad \epsilon_t \sim N\left(0, \sigma_t^2\right)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \alpha_2 \epsilon_{t-1}^2$$

where ε_t stands for the monetary policy or markup shock. The fitted volatilities of $\widehat{\sigma}_t$ for monetary and markup shocks can be obtained from the model above, after which the relative volatility of monetary and markup shocks can be constructed as $\widehat{\omega}_t = \widehat{\sigma}_{m,t} / \widehat{\sigma}_{u,t}$.

To get complete estimates of $\widehat{\omega}_t$ with the approach outlined above we need series for both shocks. However, to the best of my knowledge, there are no series of markup shocks available in the literature, hence the estimation of the volatility of markup shocks becomes infeasible. Nevertheless, in an estimated model for the US economy, Justiniano and Primiceri (2008) find that the volatility of markup shocks is relatively stable over time, whereas the volatility of monetary shocks varies significantly. Therefore, I use only estimates of monetary shocks to obtain a measure of relative volatility by assuming that the volatility of markup shocks is constant, such that $\widehat{\omega}_t \propto \widehat{\sigma}_{m,t}$.

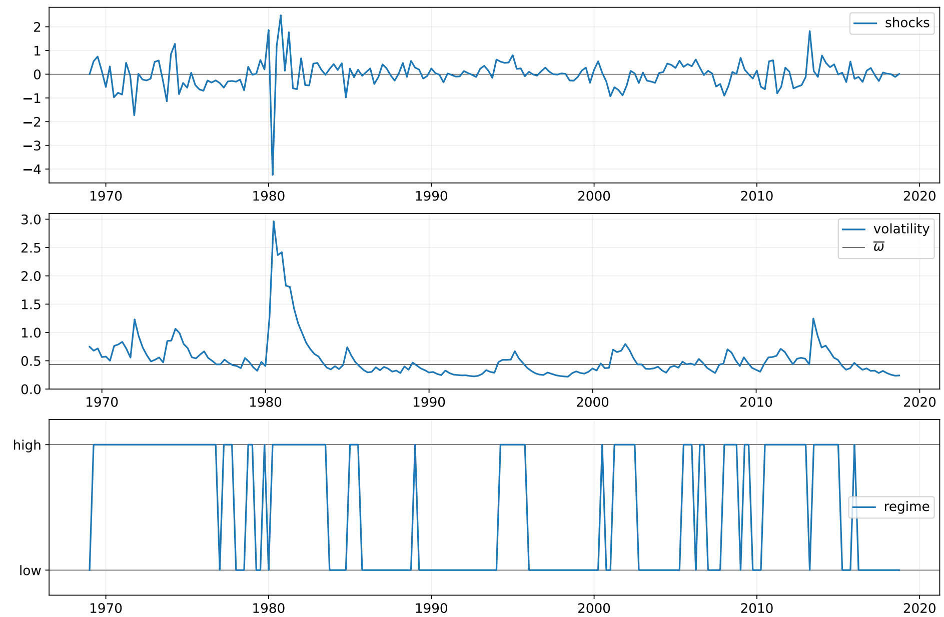


Figure 16: Volatility of monetary shocks in periods of low/high policy uncertainty in the US

Fig. 16 shows the constructed series of monetary policy shocks along with their estimated volatility. The figure also plots the periods of high and low monetary policy uncertainty based on the median volatility of 0.45 in the sample period. We can see that the volatility of monetary shocks varies substantially, being high in the 1970s and early 1980s, moderating in the 1990s and 2000s, before increasing again in the 2010s. The time-varying volatility of monetary shocks allows us to estimate the state-dependent effects of monetary policy.

5.2 Empirical analysis

Using the data described above, I estimate the effects of monetary policy shocks using the following system of equations, similar to Romer and Romer (2004)

$$\Delta y_t = a_0^y + \sum_{j=1}^{8} b_j^y \Delta y_{t-j} + \sum_{h=1}^{16} c_{L,h}^y \, \varepsilon_t^m \cdot \mathbf{1}_{\omega_t < \overline{\omega}} + \sum_{h=1}^{16} c_{H,h}^y \, \varepsilon_t^m \cdot \mathbf{1}_{\omega_t > \overline{\omega}} + \eta_t^y$$

$$\Delta p_t = a_0^p + \sum_{j=1}^{8} b_j^p \Delta p_{t-j} + \sum_{h=1}^{16} c_{L,h}^p \, \varepsilon_t^m \cdot \mathbf{1}_{\omega_t < \overline{\omega}} + \sum_{h=1}^{16} c_{H,h}^p \, \varepsilon_t^m \cdot \mathbf{1}_{\omega_t > \overline{\omega}} + \eta_t^p$$

where y_t is the log of real GDP and p_t is the log of the GDP price deflator.[21] [22]

The regression specifications above differ from the approach of Romer and Romer (2004), as I allow for different coefficients on monetary policy shocks in periods of high and low monetary policy uncertainty. Specifically, the hypotheses being tested imply $c^y_{L,h} < c^y_{H,h}$ and $c^p_{L,h} > c^p_{H,h}$ for all $h \geq 0$.

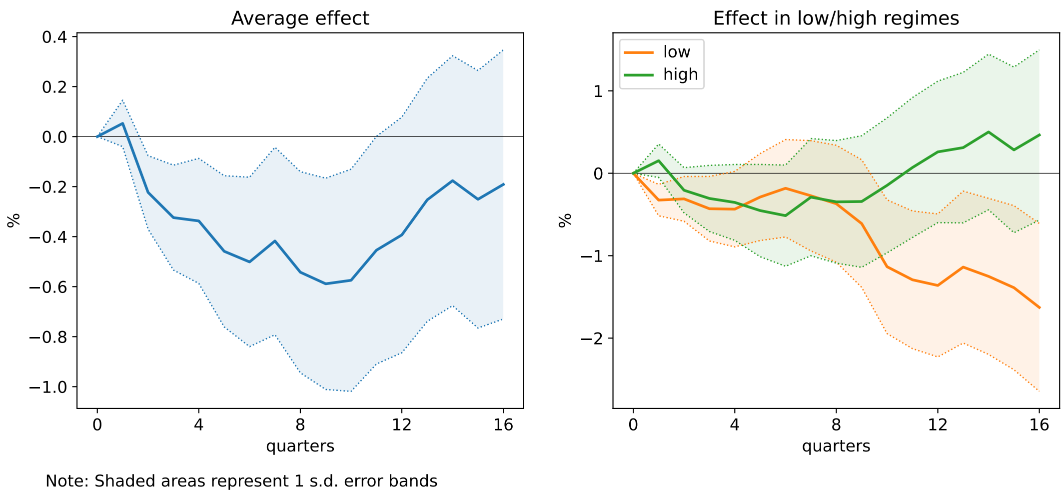


Figure 17: Effects of monetary shocks on real GDP in low/high policy uncertainty periods in the US

Fig. 17 plots the estimated responses of real GDP to a monetary policy shock in periods of high and low monetary policy uncertainty, as well as the average response estimated for the whole sample. In general, the effects of monetary policy shocks on output are stronger in periods of low monetary policy uncertainty. This result is consistent with the hypothesis $c^y_{L,h} < c^y_{H,h}$ and shows that in times of low policy uncertainty monetary shocks are likely perceived as cost-push shocks and result in greater declines in real GDP. This effect is also observed in the response of industrial production to monetary shocks, shown in the Online Appendix.

[21] In the Online Appendix, I also check the robustness of the results by using indices of industrial production, consumer and producer prices instead of real GDP and GDP deflator.

[22] Unlike Romer and Romer (2004), I do not use quarterly dummies since I use seasonally adjusted data for estimation. Nevertheless, the results are robust to using seasonally non-adjusted data and adding quarterly dummies.

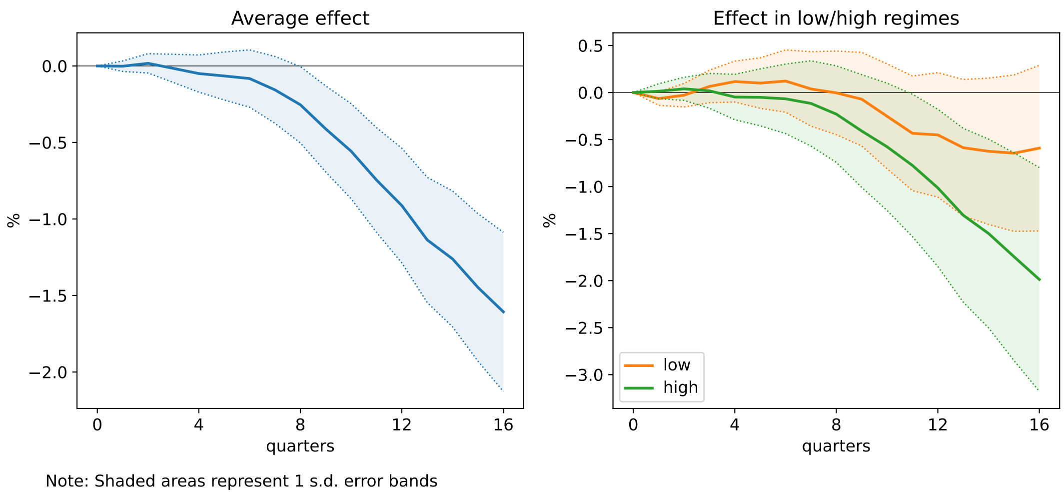


Figure 18: Effects of monetary shocks on prices in low/high policy uncertainty periods in the US

On the other hand, Fig. 18 shows that the effects of monetary policy shocks on prices are stronger in periods of high monetary policy uncertainty. This finding is consistent with the hypothesis $c^p_{L,h} > c^p_{H,h}$ and shows that in times of high policy uncertainty monetary shocks are likely correctly perceived as monetary shocks and result in greater declines in prices. In the Online Appendix, I present the responses of consumer and producer price indices to monetary shocks, respectively. The response of producer prices is similar under both regimes, while the response of consumer prices is not consistent with model predictions. This phenomenon may be explained by the fact that, unlike producer prices, the consumer price index comprises prices of goods both produced domestically and imported from abroad. Hence, the response of imported goods prices does not necessarily reflect the effects of the uncertainty channel of monetary policy, thus altering the response of the consumer price index.

The empirical results obtained in this paper show how the effects of monetary policy shocks depend on the level of policy uncertainty. However, evidence of the US Fed using the uncertainty channel of monetary policy is relatively scarce. In Ohanyan and Grigoryan (2021), we explore the empirical properties of the volatility of US monetary policy shocks. We estimate a Taylor-type monetary policy rule with time-varying heteroskedasticity to find potential determinants of the volatility of monetary shocks. Our findings suggest that the volatility of deviations from policy rule is linked to differences between headline and core inflation, which can mostly be attributed to cost-push shocks.

Conclusions

In this paper, I study the optimal monetary policy design in a New Keynesian model characterized by dispersed information and inefficient markup shocks. In this environment, the central bank aims at reducing the informativeness of interest rate changes, thereby increasing uncertainty for firms. As a potential welfare-improving feature of monetary policy, I propose a policy strategy of an interest rate rule along with systematic deviations from the implied policy rate. Deviations from the policy rule are designed to increase uncertainty about the type of shock and reduce the informativeness of the interest rate signal. I refer to this novel channel of monetary policy as the *uncertainty channel*.

Although the presence of monetary shocks reduces the effects of markup shocks on the economy, policy shocks themselves generate economic fluctuations and have direct welfare-reducing effects. Therefore, the optimality of a policy strategy involving monetary shocks depends on the relative costs and benefits of deviating from the policy rule. In the standard New Keynesian model with complete information, the uncertainty channel of monetary policy is absent, hence the policy shocks have strictly welfare-reducing effects. However, in the model with dispersed information, the benefits of monetary shocks outweigh the implied welfare costs and policy shocks have net welfare-improving effects.

The paper also tests the predictions of the model using US data. The empirical results show that the effects of monetary policy shocks on output and prices depend on the level of monetary policy uncertainty, as measured by the relative volatility of monetary and markup shocks. The effects of monetary policy shocks on output are stronger in periods of low monetary policy uncertainty, whereas the effects on prices are stronger in periods of high monetary policy uncertainty. This finding is consistent with the model predictions and shows that potential gains from the uncertainty channel are supported by empirical evidence.

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