Online Appendix for the “Country portfolios and optimal monetary policy”

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A Model solution

A.1 Households

Forming the Lagrangean yields

$$\max_{D_t, F_t, C_{t+1}, \lambda_t^y, \lambda_t^o} \quad E_t \left[\log\left(C_{t+1}\right) - \lambda_t^y \left(D_t + F_t - Q_t - B_t + T_t\right) - \right.$$
$$\left. - \lambda_{t+1}^o \left(C_{t+1} - \frac{R_t}{\Pi_{t+1}} D_t - \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}} F_t \right) \right]$$

F.O.C.

$$\begin{aligned}
[D_t]: &\quad -\lambda_t^y + E_t\left[\lambda_{t+1}^o \frac{R_t}{\Pi_{t+1}}\right] = 0 \\
[F_t]: &\quad -\lambda_t^y + E_t\left[\lambda_{t+1}^o \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}}\right] = 0 \\
[C_{t+1}]: &\quad \frac{1}{C_{t+1}} - \lambda_{t+1}^o = 0
\end{aligned}$$

Combing the optimality conditions of households above implies

$$E_t\left[\frac{1}{C_{t+1}} \frac{R_t}{\Pi_{t+1}}\right] = E_t\left[\frac{1}{C_{t+1}} \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}}\right] \tag{1}$$

Then, by substituting the budget constraint of old households into Eq. (1), it can be represented as

$$E_t\left[\frac{\frac{R_t}{\Pi_{t+1}}}{\frac{R_t}{\Pi_{t+1}} D_t + \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}} F_t}\right] = E_t\left[\frac{\Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}}}{\frac{R_t}{\Pi_{t+1}} D_t + \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}} F_t}\right]$$

or

$$E_t\left[\frac{R_t}{R_t D_t + \Psi_{t+1} R_t^* F_t}\right] = E_t\left[\frac{\Psi_{t+1} R_t^*}{R_t D_t + \Psi_{t+1} R_t^* F_t}\right]$$

Expanding the expectation operators above by using the definitions of exogenous disturbances yields

$$p \cdot \frac{R_t}{R_t D_t + \Psi_{t+1}^h R_t^* F_t} + (1-p) \cdot \frac{R_t}{R_t D_t + \Psi_{t+1}^l R_t^* F_t} =$$
$$= p \cdot \frac{\Psi_{t+1}^h R_t^*}{R_t D_t + \Psi_{t+1}^h R_t^* F_t} + (1-p) \cdot \frac{\Psi_{t+1}^l R_t^*}{R_t D_t + \Psi_{t+1}^l R_t^* F_t}$$

Then, the optimal quantity of domestic bonds is given by

$$D_t = \frac{R_t^* \Psi_{t+1}^l \Psi_{t+1}^h - R_t \left(p \Psi_{t+1}^l + (1-p) \Psi_{t+1}^h \right)}{\left(R_t - R_t^* \Psi_{t+1}^l \right) \left(R_t - R_t^* \Psi_{t+1}^h \right)} R_t^* Z_t \tag{2}$$

while the optimal quantity of foreign bonds is given by

$$F_t = \frac{R_t - R_t^* \left(p \Psi_{t+1}^h + (1-p) \Psi_{t+1}^l \right)}{\left(R_t - R_t^* \Psi_{t+1}^l \right) \left(R_t - R_t^* \Psi_{t+1}^h \right)} R_t Z_t \tag{3}$$

with

$$Z_t \equiv D_t + F_t = Q_t + B_t - T_t$$

A.2 Deterministic steady state

In a steady state with no foreign inflation shocks, we have

$$\Pi_{ss}^* = \Pi_{ss} = \Psi_{ss} = 1$$

and

$$R_{ss} = R^*$$

Then, the steady-state portfolio return will be given by

$$X_{ss} = R^*$$

Assuming that government spending is constant, i.e., $G_t = G = G_{ss}$, the government budget constraint implies

$$D_{ss} = \frac{G_{ss}}{1 + \tau - R_{ss}}$$

and

$$T_{ss} = \frac{\tau G_{ss}}{1 + \tau - R_{ss}}$$

Hence, the steady-state share of domestic bonds in the household portfolio is given by

$$\omega_{ss} \equiv \frac{D_{ss}}{Z_{ss}} = \frac{D_{ss}}{1 + B_{ss} - T_{ss}} = \frac{D_{ss}}{1 + \kappa G_{ss} - \tau D_{ss}}$$

B Extensions

B.1 Households

Forming the Lagrangean yields

$$\max_{D_t,F_t,N_t,C_{t+1},\lambda_t^y,\lambda_t^o} \quad E_t\left[\log\left(C_{t+1}\right)-\chi\frac{N_t^{1+\varphi}}{1+\varphi}-\right.$$
$$-\lambda_t^y\left(D_t+F_t-\frac{P_{T,t}}{P_t}Q_t-W_tN_t-\Omega_t-B_t+T_t\right)-$$
$$\left.-\lambda_{t+1}^o\left(C_{t+1}-\frac{R_t}{\Pi_{t+1}}D_t-\Psi_{t+1}\frac{R_t^*}{\Pi_{t+1}}F_t\right)\right]$$

F.O.C.

$$\begin{aligned}
[D_t]: &\quad -\lambda_t^y+E_t\left[\lambda_{t+1}^o\frac{R_t}{\Pi_{t+1}}\right]=0\\
[F_t]: &\quad -\lambda_t^y+E_t\left[\lambda_{t+1}^o\Psi_{t+1}\frac{R_t^*}{\Pi_{t+1}}\right]=0\\
[N_t]: &\quad -\chi N_t^{\varphi}+\lambda_t^yW_t=0\\
[C_{t+1}]: &\quad \frac{1}{C_{t+1}}-\lambda_{t+1}^o=0
\end{aligned}$$

Combining the F.O.C. for D_t and F_t, we get

$$\lambda_t^yD_t+\lambda_t^yF_t=E_t\left[\lambda_{t+1}^o\frac{R_t}{\Pi_{t+1}}D_t+\lambda_{t+1}^o\Psi_{t+1}\frac{R_t^*}{\Pi_{t+1}}F_t\right]$$

or

$$\lambda_t^y\left(D_t+F_t\right)=E_t\left[\frac{1}{C_{t+1}}\left(\frac{R_t}{\Pi_{t+1}}D_t+\Psi_{t+1}\frac{R_t^*}{\Pi_{t+1}}F_t\right)\right]$$

Hence

$$\lambda_t^y=\frac{1}{D_t+F_t}$$

Substituting the expression above into the F.O.C. of N_t yields

$$N_t^{\varphi}=\frac{1}{\chi}\frac{W_t}{Z_t}$$

where $Z_t\equiv D_t+F_t=\frac{P_{T,t}}{P_t}Q_t+W_tN_t+\Omega_t+B_t-T_t$.

B.2 Firms

Substituting the demand schedule into the firms' problem yields

$$\max_{P_{N,t}(j)} \quad E_{t-1}\left[\Lambda_{t-1,t}\left(\frac{P_{N,t}(j)}{P_t}\left(\frac{P_{N,t}(j)}{P_{N,t}}\right)^{-\epsilon}C_{N,t} - W_t\left(\frac{P_{N,t}(j)}{P_{N,t}}\right)^{-\epsilon}C_{N,t}\right)\right]$$

F.O.C.

$$E_{t-1}\left[\Lambda_{t-1,t}\left(\frac{1}{P_t}(1-\epsilon)\left(\frac{P_{N,t}(j)}{P_{N,t}}\right)^{-\epsilon}C_{N,t} + W_t\epsilon\frac{1}{P_{N,t}(j)}\left(\frac{P_{N,t}(j)}{P_{N,t}}\right)^{-\epsilon}C_{N,t}\right)\right] = 0$$

which implies

$$E_{t-1}\left[\Lambda_{t-1,t}Y_t(j)\left(\frac{P_{N,t}(j)}{P_t} - \mathcal{M}W_t\right)\right] = 0$$

or

$$E_{t-1}\left[\Lambda_{t-1,t}Y_t(j)\left(\frac{S_{N,t}(j)}{\Pi_t} - \mathcal{M}W_t\right)\right] = 0$$

where $S_{N,t} = \frac{P_{N,t}(j)}{P_{t-1}}$ and $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$.

With the employment subsidy, the optimal price-setting condition becomes

$$E_{t-1}\left[\Lambda_{t-1,t}Y_t(j)\left(\frac{S_{N,t}(j)}{\Pi_t} - W_t\right)\right] = 0$$

In a symmetric equilibrium, $Y_t(j) = Y_t$, $P_{N,t}(j) = P_{N,t}$ and $S_{N,t}(j) = S_{N,t}$ for all j, hence

$$E_{t-1}\left[\Lambda_{t-1,t}Y_t\left(\frac{S_{N,t}}{\Pi_t} - W_t\right)\right] = 0$$

Then, firms' real profits are given by

$$\Omega_t = \frac{P_{N,t}}{P_t}Y_t - (1-\nu)\,W_tN_t = \frac{S_{N,t}}{\Pi_t}Y_t - (1-\nu)\,W_tN_t$$

B.3 Aggregation

Note that the stochastic discount factor is given by

$$\Lambda_{t-1,t} = \frac{\lambda_{o,t}}{\lambda_{y,t-1}} = \frac{\frac{1}{C_t}}{\frac{1}{D_{t-1}+F_{t-1}}} = \frac{D_{t-1}+F_{t-1}}{C_t} = \frac{1}{X_t}$$

Then, the optimal price-setting condition is

$$\frac{1}{2}\left(X_t^h\right)^{-1} Y_t^h \left(\frac{S_{N,t}}{\Pi_t^h} - W_t^h\right) + \frac{1}{2}\left(X_t^l\right)^{-1} Y_t^l \left(\frac{S_{N,t}}{\Pi_t^l} - W_t^l\right) = 0$$

collecting terms

$$S_{N,t}\left(Y_t^h \left(X_t^h \Pi_t^h\right)^{-1} + Y_t^l \left(X_t^l \Pi_t^l\right)^{-1}\right) = Y_t^h \left(X_t^h\right)^{-1} W_t^h + Y_t^l \left(X_t^l\right)^{-1} W_t^l$$

Then, the optimal price for non-tradable goods is given by

$$S_{N,t} = \frac{Y_t^h W_t^h \left(X_t^h\right)^{-1} + Y_t^l W_t^l \left(X_t^l\right)^{-1}}{Y_t^h \left(X_t^h \Pi_t^h\right)^{-1} + Y_t^l \left(X_t^l \Pi_t^l\right)^{-1}}$$

Domestic CPI inflation may be written as

$$\begin{aligned}
\Pi_t &= \left((1-\gamma)\left(\frac{P_{N,t}}{P_{t-1}}\right)^{1-\eta} + \gamma\left(\frac{P_{T,t}}{P_{t-1}}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= \left((1-\gamma)\left(S_{N,t}\right)^{1-\eta} + \gamma\left(S_{T,t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}
\end{aligned}$$

where $S_{N,t} \equiv \frac{P_{N,t}}{P_{t-1}}$ and $S_{T,t} \equiv \frac{P_{T,t}}{P_{t-1}}$ are the ratios of the prices of non-tradable and tradable goods to the price level in the previous period.

We can also write

$$S_{T,t} = \frac{P_{T,t}}{P_{t-1}} = \frac{P_{T,t}}{P_{T,t-1}}\frac{P_{T,t-1}}{P_{t-2}}\frac{P_{t-2}}{P_{t-1}} = \frac{\Pi_{T,t}}{\Pi_{t-1}} S_{T,t-1}$$

On the other hand

$$\Pi_{N,t} = \frac{P_{N,t}}{P_{N,t-1}} = \frac{P_{N,t}}{P_{t-1}}\frac{P_{t-1}}{P_{t-2}}\frac{P_{t-2}}{P_{N,t-1}} = \frac{S_{N,t}}{S_{N,t-1}}\Pi_{t-1}$$

Finally, using the fact $Y_t = N_t$ and the household F.O.C.

$$W_t = \chi Y_t^{\varphi} Z_t$$

where

$$Z_t \equiv \frac{P_{T,t}}{P_t} Q_t + (1-\nu) W_t N_t + \Omega_t + B_t - T_t = \frac{S_{T,t}}{\Pi_t} Q_t + \frac{S_{N,t}}{\Pi_t} Y_t + B_t - T_t$$

B.4 Deterministic steady state

In a steady state with no foreign inflation shocks, we have

$$\Pi^*_{ss} = \Pi_{T\,ss} = \Psi_{ss} = 1$$

and

$$R_{ss} = R^*$$

I calibrate the rest of the model to target zero inflation in the steady state, i.e., $\Pi_{N\,ss} = 1$ and $\Pi_{ss} = 1$.

Then, the steady-state portfolio return will be given by

$$X_{ss} = R_{ss}$$

In the absence of uncertainty, the steady-state optimal price-setting condition becomes

$$S_{N\,ss} = W_{ss}\Pi_{ss}$$

Then, imposing $S_{N\,ss} = \Pi_{ss}$ implies that the steady-state wage has to satisfy

$$W_{ss} = 1$$

Therefore, the steady-state level of wealth should satisfy

$$Z_{ss} = \frac{1}{\chi}\frac{1}{Y_{ss}^{\varphi}}$$

On the other hand, the steady-state level of output should satisfy

$$Y_{ss} = (1-\gamma)\, C_{ss} = (1-\gamma)\, X_{ss} Z_{ss}$$

or

$$Y_{ss} = \frac{1-\gamma}{\chi}\frac{X_{ss}}{Y_{ss}^{\varphi}}$$

Thus, the steady-state level of output will be given by

$$Y_{ss} = \left(\frac{1-\gamma}{\chi} X_{ss} \right)^{\frac{1}{1+\varphi}}$$

and the wealth level will be given by

$$Z_{ss} = \frac{Y_{ss}}{(1-\gamma) X_{ss}}$$

Therefore, the endowment level must be given by

$$Q_{ss} = Z_{ss} - Y_{ss} - B_{ss} + T_{ss} = \left(\frac{1}{(1-\gamma) X_{ss}} - 1 \right) Y_{ss} - B_{ss} + T_{ss}$$

Hence, calibrating the endowment Q_{ss} to the level given by the expression above results in zero steady-state inflation.

I also calibrate the steady-state level of household income (before government transfers and taxes) to 1 for the sake of comparability with the endowment economy.[1]

This implies that $Q_{ss} + Y_{ss} = 1 = Z_{ss} - B_{ss} + T_{ss}$. Hence, to ensure zero steady-state inflation, the labor disutility parameter χ must be set to

$$\chi = Z_{ss}^{-1} Y_{ss}^{-\varphi} = (1-\gamma)^{-\varphi} X_{ss}^{-\varphi} Z_{ss}^{-1-\varphi}$$

Finally, assuming that government spending is constant, i.e., $G_t = G = G_{ss}$, the government budget constraint implies

$$D_{ss} = \frac{G_{ss}}{1 + \tau - R_{ss}}$$

and

$$T_{ss} = \frac{\tau G_{ss}}{1 + \tau - R_{ss}}$$

Hence, the steady-state share of domestic bonds in the household portfolio is given by

$$\omega_{ss} \equiv \frac{D_{ss}}{Z_{ss}} = \frac{D_{ss}}{1 + B_{ss} - T_{ss}} = \frac{D_{ss}}{1 + \kappa G_{ss} - \tau D_{ss}}$$

[1] Note that without this, different levels of country openness γ imply different steady-state levels of output Y and wealth Z.

C Proofs

C.1 Equilibrium

Lemma 1 (Existence and uniqueness of equilibrium)

Given a domestic debt level D_t, such that $0 < D_t < Z_t$, there exists a unique equilibrium interest rate R_t that satisfies the portfolio optimality conditions in Eqs. (2) and (3).

The equilibrium interest rate R_t is given by

$$R_t = \frac{1}{2} R_t^* \left[\left(\Psi_{t+1}^l + \Psi_{t+1}^h \right) - \frac{Z_t}{D_t} \left(p \Psi_{t+1}^l + (1-p) \Psi_{t+1}^h \right) + \Delta_t^{1/2} \right] \tag{4}$$

where

$$\Delta_t = \left(\Psi_{t+1}^l - \Psi_{t+1}^h \right)^2 - 2 \frac{Z_t}{D_t} \left(\Psi_{t+1}^l - \Psi_{t+1}^h \right) \left(p \Psi_{t+1}^l - (1-p) \Psi_{t+1}^h \right) + \frac{Z_t^2}{D_t^2} \left(p \Psi_{t+1}^l + (1-p) \Psi_{t+1}^h \right)^2$$

Proof (Lemma 1)

Rearranging the portfolio optimality condition in Eq. (2) and solving for R_t/R_t^* yields

$$\left(\frac{R_t}{R^*} \right)^2 - b_t \left(\frac{R_t}{R^*} \right) + c_t = 0$$

where

$$b_t = \left(\Psi_{t+1}^l + \Psi_{t+1}^h \right) - \frac{Z_t}{D_t} \left(p \Psi_{t+1}^l + (1-p) \Psi_{t+1}^h \right) \qquad \textit{and} \qquad c_t = \Psi_{t+1}^l \Psi_{t+1}^h \left(1 - \frac{Z_t}{D_t} \right)$$

Hence, the equilibrium relative interest rate is the positive root of the quadratic equation above and is given by

$$\frac{R_t}{R_t^*} = \frac{1}{2} \left(b_t + \Delta_t^{1/2} \right)$$

where

$$\Delta_t = \left(\Psi_{t+1}^l - \Psi_{t+1}^h \right)^2 - 2 \frac{Z_t}{D_t} \left(\Psi_{t+1}^l - \Psi_{t+1}^h \right) \left(p \Psi_{t+1}^l - (1-p) \Psi_{t+1}^h \right) + \frac{Z_t^2}{D_t^2} \left(p \Psi_{t+1}^l + (1-p) \Psi_{t+1}^h \right)^2$$

Then, with the use of some algebra, the solution above may be rewritten as

$$R_t = \frac{1}{2} R_t^* \left[\left(\Psi_{t+1}^l + \Psi_{t+1}^h \right) - \frac{Z_t}{D_t} \left(p \Psi_{t+1}^l + (1-p) \Psi_{t+1}^h \right) + \Delta_t^{1/2} \right]$$

Existence.

Note that the discriminant Δ_t is a quadratic function of the inverse of the share of domestic bonds in the portfolio Z_t/D_t.

Hence, Δ_t is non-negative for all values of D_t and Z_t if and only if its discriminant is non-positive. The latter is given by

$$\begin{aligned}
&4\left(\Psi^l_{t+1}-\Psi^h_{t+1}\right)^2\left(p\Psi^l_{t+1}-(1-p)\,\Psi^h_{t+1}\right)^2-4\left(p\Psi^l_{t+1}+(1-p)\,\Psi^h_{t+1}\right)^2\left(\Psi^l_{t+1}-\Psi^h_{t+1}\right)^2=\\
&=4\left(\Psi^l_{t+1}-\Psi^h_{t+1}\right)^2\left(\left(p\Psi^l_{t+1}-(1-p)\,\Psi^h_{t+1}\right)^2-\left(p\Psi^l_{t+1}+(1-p)\,\Psi^h_{t+1}\right)^2\right)\\
&=-16p\,(1-p)\,\Psi^l_{t+1}\Psi^h_{t+1}\left(\Psi^l_{t+1}-\Psi^h_{t+1}\right)^2
\end{aligned}$$

and is non-positive for all values of p, Ψ^l_{t+1}, Ψ^h_{t+1} satisfying $0<p<1$, $\Psi^l_{t+1}>0$, and $\Psi^h_{t+1}>0$.

Therefore, the equilibrium interest rate is well-defined for all values of p, Ψ^l_{t+1}, Ψ^h_{t+1} satisfying the conditions above and $0<D_t<Z_t$.

Uniqueness.

To show that the equilibrium interest rate is unique, we need to show that it is the only root of the quadratic equation above that is negative.

Note that the other solution to the quadratic equation above is given by

$$\frac{R_t}{R^*_t}=\frac{1}{2}\left(b_t-\Delta_t^{1/2}\right)$$

may imply a negative interest rate.

This root is negative if $b_t-\Delta_t^{1/2}<0$, which is equivalent to $b_t<0$ or $b_t>0$ and $b_t^2-\Delta_t<0$.

We have

$$b_t^2=\left(\Psi^l_{t+1}+\Psi^h_{t+1}\right)^2-2\frac{Z_t}{D_t}\left(\Psi^l_{t+1}+\Psi^h_{t+1}\right)\left(p\Psi^l_{t+1}+(1-p)\,\Psi^h_{t+1}\right)+\frac{Z_t^2}{D_t^2}\left(p\Psi^l_{t+1}+(1-p)\,\Psi^h_{t+1}\right)^2$$

Hence

$$b_t^2-\Delta_t=4\Psi^l_{t+1}\Psi^h_{t+1}\left(1-\frac{Z_t}{D_t}\right)<0$$

for all values of D_t and Z_t satisfying $0<D_t<Z_t$.

Therefore, the equilibrium interest rate is unique for all permissible values of p, D_t, Z_t, Ψ^l_{t+1}, Ψ^h_{t+1}. ■

Lemma 2 (First derivatives of R_t/R_t^*)
Given the equilibrium relative interest rate R_t/R_t^* as a function of the share of domestic bonds ω_t and exchange rate volatility ψ, as in Eq. (5), the following statements hold.

- *R_t/R_t^* is weakly increasing in ω_t for all $0 < \omega_t < 1$ and $\psi \geq 0$:*

$$\frac{\partial R_t/R_t^*}{\partial \omega_t} \geq 0 \qquad \textit{for} \quad 0 < \omega_t < 1$$

  with equality if and only if $\psi = 0$.

- *R_t/R_t^* is weakly decreasing in ψ for $0 < \omega_t \leq 1/2$, and weakly increasing for $1/2 \leq \omega_t < 1$ and all $\psi \geq 0$:*

$$\frac{\partial R_t/R_t^*}{\partial \psi} \leq 0 \qquad \textit{for} \quad 0 < \omega_t \leq 1/2 \qquad \textit{and} \qquad \frac{\partial R_t/R_t^*}{\partial \psi} \geq 0 \qquad \textit{for} \quad 1/2 \leq \omega_t < 1$$

  with equality if and only if $\omega_t = 1/2$.

Proof (Lemma 2)
Recall that

$$\frac{R_t}{R_t^*} = \frac{1}{2}\left[\left(e^{\psi} + e^{-\psi}\right)\left(1 - \frac{1}{2\omega_t}\right) + \Delta_t^{1/2}\right] \tag{5}$$

where

$$\Delta_t = \left(e^{\psi} - e^{-\psi}\right)^2\left(1 - \frac{1}{2\omega_t}\right)^2 + \frac{1}{\omega_t^2} \tag{6}$$

Note that

$$\frac{\partial \Delta_t}{\partial \omega_t} = \frac{1}{\omega_t^2}\left(\left(e^{\psi} - e^{-\psi}\right)^2 - \frac{1}{2\omega_t}\left(e^{\psi} + e^{-\psi}\right)^2\right) \tag{7}$$

and

$$\frac{\partial \Delta_t}{\partial \psi} = 2\left(e^{2\psi} - e^{-2\psi}\right)\left(1 - \frac{1}{2\omega_t}\right)^2 \tag{8}$$

- *Then, taking the derivative of Eq. (5) with respect to ω_t yields*

$$\frac{\partial R_t/R_t^*}{\partial \omega_t} = \frac{1}{2}\left[\left(e^{\psi} + e^{-\psi}\right)\frac{1}{2\omega_t^2} + \frac{1}{2}\Delta_t^{-1/2}\frac{\partial \Delta_t}{\partial \omega_t}\right]$$

Substituting Eq. (7) for $\partial\Delta_t/\partial\omega_t$ and rearranging terms we get

$$\frac{\partial R_t/R_t^*}{\partial\omega_t} = \frac{1}{4\omega_t^2}\left[\left(e^{\psi}+e^{-\psi}\right) + \Delta_t^{-1/2}\left(\left(e^{\psi}-e^{-\psi}\right)^2 - \frac{1}{2\omega_t}\left(e^{\psi}+e^{-\psi}\right)^2\right)\right] \tag{9}$$

or

$$\frac{\partial R_t/R_t^*}{\partial\omega_t} = \frac{1}{4\omega_t^2}\left(e^{\psi}+e^{-\psi}\right)\left[1 + \Delta_t^{-1/2}\left(\frac{\left(e^{\psi}-e^{-\psi}\right)^2}{e^{\psi}+e^{-\psi}} - \frac{1}{2\omega_t}\left(e^{\psi}+e^{-\psi}\right)\right)\right]$$

Hence, it is sufficient to show that

$$1 + \Delta_t^{-1/2}\left(\frac{\left(e^{\psi}-e^{-\psi}\right)^2}{e^{\psi}+e^{-\psi}} - \frac{1}{2\omega_t}\left(e^{\psi}+e^{-\psi}\right)\right) \geq 0$$

or equivalently

$$\Delta_t^{1/2} + \left(\frac{\left(e^{\psi}-e^{-\psi}\right)^2}{e^{\psi}+e^{-\psi}} - \frac{1}{2\omega_t}\left(e^{\psi}+e^{-\psi}\right)\right) \geq 0$$

After some algebra, the above inequality can be rewritten as

$$\Delta_t^{1/2} + \left(\frac{\left(e^{\psi}+e^{-\psi}\right)^2}{e^{\psi}+e^{-\psi}} - \frac{1}{2\omega_t}\left(e^{\psi}+e^{-\psi}\right)\right) - \frac{4}{e^{\psi}+e^{-\psi}} \geq 0$$

or

$$\Delta_t^{1/2} \geq \frac{4}{e^{\psi}+e^{-\psi}} - \left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)$$

Note that if the right-hand side of the above inequality is negative, then the inequality is satisfied.

Otherwise, substituting Eq. (6) for Δ_t, and taking squares of both sides, we can write

$$\left(e^{\psi}-e^{-\psi}\right)^2\left(1-\frac{1}{2\omega_t}\right)^2 + \frac{1}{\omega_t^2} \geq \frac{16}{\left(e^{\psi}+e^{-\psi}\right)^2} - 8\left(1-\frac{1}{2\omega_t}\right) + \left(e^{\psi}+e^{-\psi}\right)^2\left(1-\frac{1}{2\omega_t}\right)^2$$

or

$$\frac{1}{\omega_t^2} \geq \frac{16}{\left(e^{\psi}+e^{-\psi}\right)^2} - 8\left(1-\frac{1}{2\omega_t}\right) + 4\left(1-\frac{1}{2\omega_t}\right)^2$$

which simplifies to

$$\frac{16}{\left(e^{\psi}+e^{-\psi}\right)^2}-4\leq 0$$

or

$$\left(e^{\psi}+e^{-\psi}\right)^2\geq 4$$

Note that the above inequality is satisfied for any $\psi\geq 0$.

Hence

$$\frac{\partial R_t/R_t^*}{\partial\omega_t}\geq 0\qquad \textit{for}\quad 0<\omega_t<1\quad \textit{and}\quad \psi\geq 0$$

with equality if and only if $\psi=0$.

- *Similarly, taking the derivative of Eq. (5) with respect to ψ yields*

$$\frac{\partial R_t/R_t^*}{\partial\psi}=\frac{1}{2}\left[\left(e^{\psi}-e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)+\frac{1}{2}\Delta_t^{-1/2}\frac{\partial\Delta_t}{\partial\psi}\right]$$

Substituting Eq. (8) for $\partial\Delta_t/\partial\psi$ we get

$$\frac{\partial R_t/R_t^*}{\partial\psi}=\frac{1}{2}\left[\left(e^{\psi}-e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)+\Delta_t^{-1/2}\left(e^{2\psi}-e^{-2\psi}\right)\left(1-\frac{1}{2\omega_t}\right)^2\right]\tag{10}$$

The last expression can be rewritten as

$$\frac{\partial R_t/R_t^*}{\partial\psi}=\frac{1}{2}\left[\left(e^{\psi}-e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)\left(1+\Delta_t^{-1/2}\left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)\right)\right]$$

It is easy to see that the expression above is non-negative if $1/2\leq\omega_t<1$.

In case of $0<\omega_t\leq 1/2$, the expression above is non-positive if and only if

$$1+\Delta_t^{-1/2}\left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)\geq 0$$

Given that $0<\omega_t\leq 1/2$, the latter is true if and only if

$$\Delta_t^{-1}\left(e^{\psi}+e^{-\psi}\right)^2\left(\frac{1}{2\omega_t}-1\right)^2\leq 1$$

or, multiplying both sides by Δ_t *and substituting* Eq. (6) *for the latter, we get*

$$\left(e^{\psi}+e^{-\psi}\right)^2\left(\frac{1}{2\omega_t}-1\right)^2 \leq \left(e^{\psi}-e^{-\psi}\right)^2\left(1-\frac{1}{\omega_t}\right)+\frac{1}{4\omega_t^2}\left(e^{\psi}+e^{-\psi}\right)^2$$

We can write the inequality above as

$$\left(e^{\psi}+e^{-\psi}\right)^2\left(\frac{1}{2\omega_t}-1\right)^2 \leq \left(e^{\psi}+e^{-\psi}\right)^2\left(\frac{1}{2\omega_t}-1\right)^2+4\left(\frac{1}{\omega_t}-1\right)$$

Therefore, noting that $0<\omega_t\leq 1/2$ *implies* $\left(\frac{1}{\omega_t}-1\right)\geq 1$*, we can see that the inequality is true for all* $0<\omega_t\leq 1/2$*.*

Hence, for all $\psi \geq 0$

$$\frac{\partial R_t/R_t^*}{\partial\psi}\leq 0 \qquad \text{for} \quad 0<\omega_t\leq 1/2 \qquad \text{and} \qquad \frac{\partial R_t/R_t^*}{\partial\psi}\geq 0 \qquad \text{for} \quad 1/2\leq\omega_t<1$$

with equality if and only if $\omega_t = 1/2$*.*

■

C.2 Welfare

Lemma 3 (Determinants of expected log returns)

The following statements hold for the expected log portfolio return.

- *The expected log portfolio return is weakly increasing in the share of domestic bonds:*

$$\frac{\partial}{\partial \omega_t} E_t \left[\log\left(X_{t+1}\right)\right] \geq 0 \qquad \text{for} \quad 0 < \omega_t < 1 \quad \text{and} \quad \phi \leq 1$$

  with equality if and only if $\phi = 1$.

- *The expected log portfolio return is weakly decreasing in the monetary policy strategy parameter:*

$$\frac{\partial}{\partial \phi} E_t \left[\log\left(X_{t+1}\right)\right] \leq 0 \qquad \text{for} \quad 0 < \omega_t < 1 \quad \text{and} \quad \phi \leq 1$$

  with equality if and only if $\phi = 1$.

Proof (Lemma 3)

Recall that

$$X_{t+1} = \omega_t \frac{R_t}{\Pi_{t+1}} + \left(1 - \omega_t\right) \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}}$$

- *We have*

$$\frac{\partial}{\partial \omega_t} E_t \left[\log\left(X_{t+1}\right)\right] = E_t \left[\frac{1}{X_{t+1}} \frac{\partial X_{t+1}}{\partial \omega_t}\right]$$

  and

$$\frac{\partial X_{t+1}}{\partial \omega_t} = \frac{R_t}{\Pi_{t+1}} - \Psi_{t+1} \frac{R_t^*}{\Pi_{t+1}} + \omega_t \frac{1}{\Pi_{t+1}} \frac{\partial R_t}{\partial \omega_t}$$

  Hence,

$$\begin{aligned} \frac{\partial}{\partial \omega_t} E_t \left[\log\left(X_{t+1}\right)\right] &= E_t \left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}} \left(R_t - \Psi_{t+1} R_t^* + \omega_t \frac{\partial R_t}{\partial \omega_t}\right)\right] \\ &= \omega_t \frac{\partial R_t}{\partial \omega_t} E_t \left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}}\right] + E_t \left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}} \left(R_t - \Psi_{t+1} R_t^*\right)\right] \end{aligned}$$

We have

$$\frac{1}{X_{t+1}}\frac{1}{\Pi_{t+1}}=\frac{1}{\omega_t\frac{R_t}{\Pi_{t+1}}+(1-\omega_t)\,\Psi_{t+1}\frac{R_t^*}{\Pi_{t+1}}}\frac{1}{\Pi_{t+1}}=\frac{1}{\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}R_t^*}$$

Therefore, the first expectation is

$$E_t\left[\frac{1}{\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}R_t^*}\right]=\frac{1}{2}\frac{2\omega_t R_t+(1-\omega_t)\left(\Psi_{t+1}^l+\Psi_{t+1}^h\right)R_t^*}{\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^l R_t^*\right)}\geq 0$$

and the second expectation is

$$\begin{aligned}
&E_t\left[\frac{R_t-\Psi_{t+1}R_t^*}{\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}R_t^*}\right]=\\
&=\frac{1}{2}\frac{\left(R_t-\Psi_{t+1}^h R_t^*\right)\left(\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^l R_t^*\right)\right)}{\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^l R_t^*\right)}\\
&+\frac{1}{2}\frac{\left(R_t-\Psi_{t+1}^l R_t^*\right)\left(\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^h R_t^*\right)\right)}{\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^l R_t^*\right)}\\
&=\frac{\omega_t\left(R_t-\Psi_{t+1}^h R_t^*\right)\left(R_t-\Psi_{t+1}^l R_t^*\right)}{\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^l R_t^*\right)}+\\
&+\frac{1}{2}\frac{R_t R_t^*\left(\Psi_{t+1}^h+\Psi_{t+1}^l\right)-2R_t^*R_t^*\Psi_{t+1}^l\Psi_{t+1}^h}{\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^l R_t^*\right)}
\end{aligned}$$

Substituting Eq. (2) for ω_t

$$\begin{aligned}
&E_t\left[\frac{R_t-\Psi_{t+1}R_t^*}{\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}R_t^*}\right]=\\
&=\frac{1}{2}\frac{2R_t^*R_t^*\Psi_{t+1}^l\Psi_{t+1}^h-R_t R_t^*\left(\Psi_{t+1}^l+\Psi_{t+1}^h\right)}{\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^l R_t^*\right)}+\\
&+\frac{1}{2}\frac{R_t R_t^*\left(\Psi_{t+1}^h+\Psi_{t+1}^l\right)-2R_t^*R_t^*\Psi_{t+1}^l\Psi_{t+1}^h}{\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+(1-\omega_t)\,\Psi_{t+1}^l R_t^*\right)}\\
&=0
\end{aligned}$$

Hence, recalling that Lemma 2 implies that

$$\frac{\partial R_t}{\partial\omega_t}=R_t^*\frac{\partial R_t/R_t^*}{\partial\omega_t}\geq 0$$

we have

$$\frac{\partial}{\partial \omega_t} E_t \left[\log\left(X_{t+1}\right)\right] = \omega_t \frac{\partial R_t}{\partial \omega_t} E_t \left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}}\right] \geq 0$$

Therefore,

$$\frac{\partial}{\partial \omega_t} E_t \left[\log\left(X_{t+1}\right)\right] \geq 0 \qquad \textit{for} \quad 0 < \omega_t < 1 \quad \textit{and} \quad \phi \leq 1$$

with equality if and only if $\phi = 1$*.*

- *We have*

$$\frac{\partial}{\partial \phi} E_t \left[\log\left(X_{t+1}\right)\right] = E_t \left[\frac{1}{X_{t+1}} \frac{\partial X_{t+1}}{\partial \phi}\right]$$

and

$$\frac{\partial X_{t+1}}{\partial \phi} = \omega_t \left(\frac{\partial R_t}{\partial \phi} \frac{1}{\Pi_{t+1}} - \frac{R_t}{\Pi_{t+1}^2} \frac{\partial \Pi_{t+1}}{\partial \phi}\right)$$

Using the definition of Π_{t+1} *we get*

$$\frac{\partial \Pi_{t+1}}{\partial \phi} = {\Pi_{t+1}^*}^{\phi} \log \Pi_{t+1}^* = \pi_{t+1}^* \Pi_{t+1}$$

On the other hand, using the relationship between ϕ *and* ψ *we have*

$$\frac{\partial R_t}{\partial \phi} = R_t^* \frac{\partial R_t / R^*}{\partial \psi} \frac{\partial \psi}{\partial \phi} = -\sigma_{\pi^*} R^* \frac{\partial R_t / R^*}{\partial \psi}$$

Therefore

$$\frac{\partial X_{t+1}}{\partial \phi} = \omega_t \left(-\sigma_{\pi^*} R^* \frac{\partial R_t / R^*}{\partial \psi} \frac{1}{\Pi_{t+1}} - \frac{R_t}{\Pi_{t+1}} \pi_{t+1}^*\right)$$

Hence,

$$\begin{aligned}
\frac{\partial}{\partial \phi} E_t \left[\log\left(X_{t+1}\right)\right] &= -\omega_t E_t \left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}} \left(\sigma_{\pi^*} R_t^* \frac{\partial R_t / R^*}{\partial \psi} + \pi_{t+1}^* R_t\right)\right] \\
&= -\omega_t \sigma_{\pi^*} R_t^* \frac{\partial R_t / R^*}{\partial \psi} E_t \left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}}\right] - \omega_t R_t E_t \left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}} \pi_{t+1}^*\right]
\end{aligned}$$

We have

$$\frac{1}{X_{t+1}}\frac{1}{\Pi_{t+1}}=\frac{1}{\omega_t\frac{R_t}{\Pi_{t+1}}+\left(1-\omega_t\right)\Psi_{t+1}\frac{R_t^*}{\Pi_{t+1}}}\frac{1}{\Pi_{t+1}}=\frac{1}{\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}R_t^*}$$

Therefore, the first expectation is

$$E_t\left[\frac{1}{\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}R_t^*}\right]=\frac{1}{2}\frac{2\omega_t R_t+\left(1-\omega_t\right)\left(\Psi_{t+1}^l+\Psi_{t+1}^h\right)R_t^*}{\left(\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}^l R_t^*\right)}$$

and the second expectation is

$$E_t\left[\frac{\pi_{t+1}^*}{\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}R_t^*}\right]=\frac{\sigma_{\pi^*}}{2}\frac{\left(1-\omega_t\right)R_t^*\left(\Psi_{t+1}^l-\Psi_{t+1}^h\right)}{\left(\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}^l R_t^*\right)}$$

Hence

$$\begin{aligned}\frac{\partial}{\partial\phi}E_t\left[\log\left(X_{t+1}\right)\right]=-\omega_t\sigma_{\pi^*}\frac{R_t^*}{2}&\frac{1}{\left(\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}^h R_t^*\right)\left(\omega_t R_t+\left(1-\omega_t\right)\Psi_{t+1}^l R_t^*\right)}\times\\ &\times\left[\left[2\omega_t R_t+\left(1-\omega_t\right)\left(\Psi_{t+1}^l+\Psi_{t+1}^h\right)R_t^*\right]\frac{\partial R_t/R^*}{\partial\psi}+\left(1-\omega_t\right)\left(\Psi_{t+1}^l-\Psi_{t+1}^h\right)R_t\right]\end{aligned}$$

Given that $\sigma_{\pi^*}\geq 0$, *and* $0<\omega_t<1$

$$\frac{\partial}{\partial\phi}E_t\left[\log\left(X_{t+1}\right)\right]\leq 0$$

if and only if the last term in the square brackets above is non-negative.

We can note that the sign of the latter depends on the sign of $\frac{\partial R_t/R^*}{\partial\psi}$, *since* $\Psi_{t+1}^l=e^{\psi}$, $\Psi^h=e^{-\psi}$, *with* $\psi\geq 0$, *therefore,* $\Psi_{t+1}^l\geq\Psi_{t+1}^h$.

If $1/2\leq\omega_t<1$, *then Lemma 2 implies that*

$$\frac{\partial R_t/R_t^*}{\partial\psi}\geq 0$$

and the last term in the square brackets above is non-negative.

If $0<\omega_t\leq 1/2$, *then Lemma 6 implies that*

$$-2\frac{e^{\psi}-e^{-\psi}}{\left(e^{\psi}+e^{-\psi}\right)^2}<\frac{\partial R_t/R^*}{\partial\psi}\leq 0$$

for all $0\leq\psi\leq 1$, *which is satisfied for the parameter values considered.*

On the other hand, if $0 < \omega_t \leq 1/2$, then Lemma 2 implies that $R_t \leq R^*$.

Using the results above, we can see that the last term in the square brackets above satisfies

$$
\begin{aligned}
&\left[2\omega_t R_t + (1-\omega_t)\left(\Psi^l_{t+1} + \Psi^h_{t+1}\right) R^*_t\right] \frac{\partial R_t / R^*}{\partial \psi} + (1-\omega_t)\left(\Psi^l_{t+1} - \Psi^h_{t+1}\right) R_t > \\
&> -\frac{1}{2}\frac{\left(e^{\psi} - e^{-\psi}\right)}{\left(e^{\psi} + e^{-\psi}\right)^2}\left[2\omega_t R_t + (1-\omega_t)\left(e^{\psi} + e^{-\psi}\right) R^*_t\right] + (1-\omega_t)\left(e^{\psi} - e^{-\psi}\right) R_t \\
&= \left(e^{\psi} - e^{-\psi}\right)\left[(1-\omega_t) R_t - \frac{1}{\left(e^{\psi} + e^{-\psi}\right)}\left(\frac{1}{\left(e^{\psi} + e^{-\psi}\right)}\omega_t R_t + (1-\omega_t)\frac{1}{2}R^*_t\right)\right]
\end{aligned}
$$

Recalling that Lemma 5 implies that

$$\frac{2}{e^{\psi} + e^{-\psi}} \leq \frac{R_t}{R^*_t} \leq \frac{1}{2}\left(e^{\psi} + e^{-\psi}\right)$$

from which we get

$$(1-\omega_t) R_t \geq (1-\omega_t)\frac{2}{e^{\psi} + e^{-\psi}} R^*_t$$

and

$$\frac{1}{\left(e^{\psi} + e^{-\psi}\right)}\left(\frac{1}{\left(e^{\psi} + e^{-\psi}\right)}\omega_t R_t + (1-\omega_t)\frac{1}{2}R^*_t\right) \leq \frac{1}{\left(e^{\psi} + e^{-\psi}\right)}\left(\frac{1}{2}\omega_t R^*_t + (1-\omega_t)\frac{1}{2}R^*_t\right)$$

Hence, for all $0 < \omega_t \leq 1/2$, we have

$$
\begin{aligned}
&\left(e^{\psi} - e^{-\psi}\right)\left[(1-\omega_t) R_t - \frac{1}{\left(e^{\psi} + e^{-\psi}\right)}\left(\frac{1}{\left(e^{\psi} + e^{-\psi}\right)}\omega_t R_t + (1-\omega_t)\frac{1}{2}R^*_t\right)\right] \\
&\geq \left(e^{\psi} - e^{-\psi}\right)\left[(1-\omega_t)\frac{2}{e^{\psi} + e^{-\psi}}R^*_t - \frac{1}{\left(e^{\psi} + e^{-\psi}\right)}\left(\frac{1}{2}\omega_t R^*_t + (1-\omega_t)\frac{1}{2}R^*_t\right)\right] \\
&= 2\frac{e^{\psi} - e^{-\psi}}{e^{\psi} + e^{-\psi}} R^* \left[(1-\omega_t) - \frac{1}{4}\right] \geq 0
\end{aligned}
$$

Therefore,

$$\frac{\partial}{\partial \phi} E_t\left[\log\left(X_{t+1}\right)\right] \leq 0 \qquad \textit{for} \quad 0 < \omega_t < 1 \quad \textit{and} \quad \phi \leq 1$$

with equality if and only if $\phi = 1$.

■

Proposition 1 (Monetary policy and portfolio returns)
The following statements hold for the mean portfolio allocation effect, i.e., the average portfolio return effect across all generations.

- *The mean portfolio return effect is non-negative:*

$$E\left[\frac{\partial}{\partial\phi}E_t\left[\log\left(X_{t+1}\right)\right]\right] \leq 0$$

with equality if and only if $\phi = 1$.

Proof (Proposition 1)

- *Recall from Lemma 3 that*

$$\frac{\partial}{\partial\phi}E_t\left[\log\left(X_{t+1}\right)\right] \leq 0 \qquad \textit{for} \quad 0 < \omega_t < 1 \quad \textit{and} \quad \phi \leq 1$$

with equality if and only if $\phi = 1$.

Then, it is straightforward to see that

$$E\left[\frac{\partial}{\partial\phi}E_t\left[\log\left(X_{t+1}\right)\right]\right] \leq 0$$

with equality if and only if $\phi = 1$.

■

Proposition 2 (Monetary policy and portfolio allocation)
The following statements hold for the mean portfolio allocation effect, i.e., the average portfolio allocation effect across all generations.

- *Under an exchange rate peg, i.e., at $\phi = 1$, the mean portfolio allocation effect is zero:*

$$E\left[\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right] \frac{d\omega_t}{d\phi}\right]\Bigg|_{\phi=1} = 0$$

- *Under inflation targeting, i.e., at $\phi = 0$, such that the economy is at its long-run equilibrium,*

  – *the mean portfolio allocation effect is non-negative if $0 < \overline{\omega} \leq 1/2$:*

$$E\left[\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right] \frac{d\omega_t}{d\phi}\right]\Bigg|_{\phi=0} \geq 0 \qquad \text{if} \quad 0 < \overline{\omega} \leq 1/2$$

  – *the mean portfolio allocation effect is non-positive if $1/2 \leq \overline{\omega} < 1$:*

$$E\left[\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right] \frac{d\omega_t}{d\phi}\right]\Bigg|_{\phi=0} \leq 0 \qquad \text{if} \quad 1/2 \leq \overline{\omega} < 1$$

  with equality if and only if $\overline{\omega} = 1/2$.

Proof (Proposition 2)
Recall from the proof of Lemma 3, that we have

$$\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right] = \omega_t \frac{\partial R_t}{\partial \omega_t} E_t\left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}}\right] = \omega_t R_t^* \frac{\partial R_t / R_t^*}{\partial \omega_t} E_t\left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}}\right]$$

On the other hand, using the definition of ω_t and Z_t we have

$$\frac{d\omega_t}{d\phi} = \frac{dD_t/Z_t}{d\phi} = \frac{dD_t}{d\phi}\frac{1}{Z_t} - \frac{D_t}{Z_t^2}\frac{dZ_t}{d\phi} = \frac{1}{Z_t}\left(\frac{dD_t}{d\phi} - \omega_t \frac{dZ_t}{d\phi}\right) = \frac{1}{Z_t}\left(\frac{dD_t}{d\phi} + \tau\omega_t \frac{dD_{t-1}}{d\phi}\right)$$

Therefore, by combining the results above, we can write

$$E\left[\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right] \frac{d\omega_t}{d\phi}\right] = E\left[\frac{\omega_t R_t^*}{Z_t} \frac{\partial R_t / R_t^*}{\partial \omega_t} E_t\left[\frac{1}{X_{t+1}} \frac{1}{\Pi_{t+1}}\right]\left(\frac{dD_t}{d\phi} + \tau\omega_t \frac{dD_{t-1}}{d\phi}\right)\right] \quad (11)$$

- *Under an exchange rate peg we have $\phi = 1$, and by Lemma 3, we have*

$$\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right] = 0$$

Therefore

$$E\left[\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right]\frac{d\omega_t}{d\phi}\right] = 0$$

- *Under inflation targeting, we have* $\phi = 0$.

  Note that since under inflation targeting the economy is in the long-run equilibrium, then $\omega_t = \overline{\omega}$, $Z_t = \overline{Z}$, *and* $\Pi_t = 1$ *are constant for all* t. *Also, by Lemma 2, we have that* $\frac{\partial R_t/R_t^*}{\partial \omega_t}$ *is a positive constant under inflation targeting.*

  On the other hand, note that under inflation targeting the inner expectation term in Eq. (11) is a positive constant, since it depends on ω_t, R_t, R_t^*, *which are constant under inflation targeting, and* Π_{t+1}, Ψ_{t+1}, *which are independent of* t.

  Thus, the sign of Eq. (11) depends on the sign of $E\left[\frac{dD_t}{d\phi}\right] = E\left[\frac{dD_{t-1}}{d\phi}\right] = \frac{dE[D_t]}{d\phi}$.

  By Lemma 8, under inflation targeting we have

$$\left.\frac{dE\left[D_t\right]}{d\phi}\right|_{\phi=0} \geq 0 \qquad \textit{if} \quad 0 < \overline{\omega} \leq 1/2$$

  and

$$\left.\frac{dE\left[D_t\right]}{d\phi}\right|_{\phi=0} \leq 0 \qquad \textit{if} \quad 1/2 \leq \overline{\omega} < 1$$

  with equality if and only if $\overline{\omega} = 1/2$.

  Therefore

$$\left.E\left[\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right]\frac{d\omega_t}{d\phi}\right]\right|_{\phi=0} \geq 0 \qquad \textit{if} \quad 0 < \overline{\omega} \leq 1/2$$

  and

$$\left.E\left[\frac{\partial}{\partial \omega_t} E_t\left[\log\left(X_{t+1}\right)\right]\frac{d\omega_t}{d\phi}\right]\right|_{\phi=0} \leq 0 \qquad \textit{if} \quad 1/2 \leq \overline{\omega} < 1$$

■

Proposition 3 (Monetary policy and household wealth)
The following statements hold for the mean wealth effect, i.e., the average wealth effect across all generations.

- *Under an exchange rate peg, i.e., at $\phi = 1$, the mean wealth effect is non-positive:*

$$E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right]\Big|_{\phi=1} \leq 0$$

  with equality if and only if $\sigma_{\pi^*} = 0$.

- *Under inflation targeting, i.e., at $\phi = 0$, such that the economy is at its long-run equilibrium,*

  - *the mean wealth effect is non-positive if $0 < \overline{\omega} \leq 1/2$:*

$$E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right]\Big|_{\phi=0} \leq 0 \qquad \text{if} \quad 0 < \overline{\omega} \leq 1/2$$

  - *the mean wealth effect is non-negative if $1/2 \leq \overline{\omega} < 1$:*

$$E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right]\Big|_{\phi=0} \geq 0 \qquad \text{if} \quad 1/2 \leq \overline{\omega} < 1$$

  with equality if and only if $\overline{\omega} = 1/2$.

Proof (Proposition 3)
Recall that

$$\frac{d}{d\phi}\log\left(Z_t\right) = \frac{1}{Z_t}\frac{dZ_t}{d\phi} = -\tau\frac{1}{Z_t}\frac{dD_{t-1}}{d\phi}$$

Therefore

$$E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right] = -\tau E\left[\frac{1}{Z_t}\frac{dD_{t-1}}{d\phi}\right]$$

Also, recall that from Lemma 7 we have

$$\frac{dD_t}{d\phi} = \frac{d\Lambda_t}{d\phi}D_{t-1} + \Lambda_t\frac{d\Lambda_{t-1}}{d\phi}D_{t-2} + \Lambda_t\Lambda_{t-1}\frac{d\Lambda_{t-2}}{d\phi}D_{t-3} + \ldots + \Lambda_t\Lambda_{t-1}\ldots\Lambda_2\frac{d\Lambda_1}{d\phi}D_0$$

where $\Lambda_t \equiv \frac{R_{t-1}}{\Pi_t} - \tau$.

Then

$$E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right] = -\tau E\left[\frac{1}{Z_t}\left(\frac{d\Lambda_{t-1}}{d\phi}D_{t-2} + \Lambda_{t-1}\frac{d\Lambda_{t-2}}{d\phi}D_{t-3} + \right.\right. \\ \left.\left. + \ldots + \Lambda_{t-1}\Lambda_{t-2}\ldots\Lambda_2\frac{d\Lambda_1}{d\phi}D_0\right)\right] \tag{12}$$

Also, note that

$$\frac{d\Lambda_t}{d\phi} = \frac{dR_{t-1}}{d\phi}\frac{1}{\Pi_t} - \frac{R_{t-1}}{\Pi_t^2}\frac{d\Pi_t}{d\phi} = -\sigma_{\pi^*}R^*_{t-1}\frac{dR_{t-1}/R^*_{t-1}}{d\psi}\frac{1}{\Pi_t} - R_{t-1}\frac{\pi^*_t}{\Pi_t}$$

- *Under an exchange rate peg, we have* $\phi = 1$ *and* $R_t = R^*_t$.

  Therefore

$$\Lambda_t\big|_{\phi=1} = \frac{R^*_{t-1}}{\Pi^*_t} - \tau \qquad\qquad \left.\frac{d\Lambda_t}{d\phi}\right|_{\phi=1} = -R^*_{t-1}\frac{\pi^*_t}{\Pi^*_t}$$

  Note that since foreign interest rate is fixed, $R^*_t = R^*$, *and foreign inflation* π^*_t *is assumed to be IID, then* Λ_t *and* $\frac{d\Lambda_t}{d\phi}$ *are IID, too.*

  Therefore, Eq. (12) under an exchange rate peg becomes

$$\left.E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right]\right|_{\phi=1} = -\tau\left(E\left[\frac{d\Lambda_{t-1}}{d\phi}\right]E\left[\frac{D_{t-2}}{Z_t}\right] + E\left[\Lambda_{t-1}\right]E\left[\frac{d\Lambda_{t-2}}{d\phi}\right]E\left[\frac{D_{t-3}}{Z_t}\right] + \right. \\ \left. + \ldots + E\left[\Lambda_{t-1}\right]E\left[\Lambda_{t-2}\right]\ldots E\left[\Lambda_2\right]E\left[\frac{d\Lambda_1}{d\phi}\right]E\left[\frac{D_0}{Z_t}\right]\right) \tag{13}$$

  On the other hand, note that

$$\left.E\left[\frac{d\Lambda_t}{d\phi}\right]\right|_{\phi=1} = \frac{\sigma_{\pi^*}}{2}R^*_{t-1}\left(e^{\sigma_{\pi^*}} - e^{-\sigma_{\pi^*}}\right) > 0$$

  Therefore, provided that $Z_t > 0$, $\Lambda_t > 0$ *and* $D_t > 0$ *for all* t, *then all terms in Eq. (13) are positive, and*

$$\left.E\left[\frac{d}{d\phi}\log\left(Z_t\right)\right]\right|_{\phi=1} \leq 0$$

  with equality if and only if $\sigma_{\pi^*} = 0$.

- *Under inflation targeting, we have* $\phi = 0$*, then* $\pi_t = 0$*, and*

$$\left.\frac{d\Lambda_t}{d\phi}\right|_{\phi=0} = -\sigma_{\pi^*} R^*_{t-1} \frac{dR_{t-1}/R^*_{t-1}}{d\psi} - \pi^*_t R_{t-1}$$

Note that since under inflation targeting the economy is in the long-run equilibrium, then $R_t = \overline{R}$*,* $\Lambda_t = \overline{\Lambda}$*,* $Z_t = \overline{Z}$ *and* $D_t = \overline{D}$ *are constant for all* t*. Also, since foreign interest rate is fixed,* $R^*_t = R^*$*, and foreign inflation* π^*_t *is assumed to be IID, then* $\frac{d\Lambda_t}{d\phi}$ *is IID, too.*

Therefore, Eq. (12) under inflation targeting is

$$E\left[\frac{d}{d\phi}\log(Z_t)\right]\Bigg|_{\phi=0} = -\tau\left(E\left[\frac{d\Lambda_{t-1}}{d\phi}\right]\frac{\overline{D}}{\overline{Z}} + \overline{\Lambda}E\left[\frac{d\Lambda_{t-2}}{d\phi}\right]\frac{\overline{D}}{\overline{Z}} + \ldots + \overline{\Lambda}^{t-1}E\left[\frac{d\Lambda_1}{d\phi}\right]\frac{\overline{D}}{\overline{Z}}\right)$$

On the other hand, note that since $E[\pi^*_t] = 0$*, then*

$$E\left[\frac{d\Lambda_t}{d\phi}\right]\Bigg|_{\phi=0} = -\sigma_{\pi^*} R^* \frac{dR_{t-1}/R^*_{t-1}}{d\psi}$$

Also, recall that Lemma 2 implies that

$$\frac{\partial R_t/R^*_t}{\partial\psi} \leq 0 \qquad \text{if} \quad 0 < \omega_t \leq 1/2 \qquad \text{and} \qquad \frac{\partial R_t/R^*_t}{\partial\psi} \geq 0 \qquad \text{if} \quad 1/2 \leq \omega_t < 1$$

with equality if and only if $\omega_t = 1/2$*.*

Therefore, provided that $\overline{Z} > 0$*,* $\overline{\Lambda} > 0$ *and* $\overline{D} > 0$*, then*

$$E\left[\frac{d}{d\phi}\log(Z_t)\right]\Bigg|_{\phi=0} \leq 0 \qquad \text{if} \quad 0 < \overline{\omega} \leq 1/2$$

and

$$E\left[\frac{d}{d\phi}\log(Z_t)\right]\Bigg|_{\phi=0} \geq 0 \qquad \text{if} \quad 1/2 \leq \overline{\omega} < 1$$

with equality if and only if $\overline{\omega} = 1/2$*.*

■

D Additional theoretical results

Lemma 4 (Second derivatives of R_t/R_t^*)
Given the equilibrium relative interest rate R_t/R_t^* as a function of the share of domestic bonds ω_t and exchange rate volatility ψ, as in Eq. (5), the following statements hold.

- *The sensitivity of R_t/R_t^* to ω_t is weakly increasing in ψ for all $0 < \omega_t < 1$ and $0 \leq \psi \leq 1$:*

$$\frac{\partial^2 R_t/R_t^*}{\partial\omega_t\partial\psi} \geq 0 \qquad \textit{for} \quad 0 < \omega_t < 1 \quad \textit{and} \quad 0 \leq \psi \leq 1$$

with equality if and only if $\psi = 0$.

Proof (Lemma 4)
Recall that

$$\frac{R_t}{R_t^*} = \frac{1}{2}\left[\left(e^{\psi} + e^{-\psi}\right)\left(1 - \frac{1}{2\omega_t}\right) + \Delta_t^{1/2}\right]$$

where

$$\Delta_t = \left(e^{\psi} - e^{-\psi}\right)^2\left(1 - \frac{1}{2\omega_t}\right)^2 + \frac{1}{\omega_t^2}$$

Then, taking the first derivatives of Δ_t with respect to ω_t and ψ, we have

$$\frac{\partial\Delta_t}{\partial\omega_t} = \frac{1}{\omega_t^2}\left(\left(e^{\psi} - e^{-\psi}\right)^2 - \frac{1}{2\omega_t}\left(e^{\psi} + e^{-\psi}\right)^2\right)$$

and

$$\frac{\partial\Delta_t}{\partial\psi} = 2\left(e^{2\psi} - e^{-2\psi}\right)\left(1 - \frac{1}{2\omega_t}\right)^2$$

Then, taking the second derivatives of Δ_t, we have

$$\frac{\partial^2\Delta_t}{\partial\omega_t^2} = \frac{3}{2\omega_t^4}\left(e^{\psi} + e^{-\psi}\right)^2 - \frac{2}{\omega_t^3}\left(e^{\psi} - e^{-\psi}\right)^2$$

and

$$\frac{\partial^2\Delta_t}{\partial\omega_t\partial\psi} = \frac{2}{\omega_t^2}\left(1 - \frac{1}{2\omega_t}\right)\left(e^{2\psi} - e^{-2\psi}\right)$$

and

$$\frac{\partial^2 \Delta_t}{\partial \psi^2} = 4\left(e^{2\psi} + e^{-2\psi}\right)\left(1 - \frac{1}{2\omega_t}\right)^2$$

- *Taking the derivative of Eq. (9) with respect to ψ yields*

$$\frac{\partial^2 R_t/R_t^*}{\partial \omega_t \partial \psi} = \frac{1}{2}\left[\left(e^{\psi} - e^{-\psi}\right)\frac{1}{2\omega_t^2} - \frac{1}{4}\Delta_t^{-3/2}\frac{\partial \Delta_t}{\partial \psi}\frac{\partial \Delta_t}{\partial \omega_t} + \frac{1}{2}\Delta^{-1/2}\frac{\partial^2 \Delta_t}{\partial \omega_t \partial \psi}\right]$$

or

$$\frac{\partial^2 R_t/R_t^*}{\partial \omega_t \partial \psi} = \frac{1}{2}\left[\left(e^{\psi} - e^{-\psi}\right)\frac{1}{2\omega_t^2} - \frac{1}{2}\Delta_t^{-1/2}\left(\frac{1}{2}\Delta_t^{-1}\frac{\partial \Delta_t}{\partial \psi}\frac{\partial \Delta_t}{\partial \omega_t} - \frac{\partial^2 \Delta_t}{\partial \omega_t \partial \psi}\right)\right] \tag{14}$$

The last term in brackets is

$$\frac{1}{2}\Delta_t^{-1}\left(2\left(e^{2\psi} - e^{-2\psi}\right)\left(1 - \frac{1}{2\omega_t}\right)^2\right)\left(\frac{1}{\omega_t^2}\left(e^{\psi} - e^{-\psi}\right)^2 - \frac{1}{2\omega_t^3}\left(e^{\psi} + e^{-\psi}\right)^2\right) - \\ - \frac{2}{\omega_t^2}\left(1 - \frac{1}{2\omega_t}\right)\left(e^{2\psi} - e^{-2\psi}\right)$$

or

$$\frac{1}{\omega_t^2}\left(e^{2\psi} - e^{-2\psi}\right)\left(1 - \frac{1}{2\omega_t}\right) \times \\ \times \left(\Delta_t^{-1}\left(1 - \frac{1}{2\omega_t}\right)\left(\left(e^{\psi} - e^{-\psi}\right)^2 - \frac{1}{2\omega_t}\left(e^{\psi} + e^{-\psi}\right)^2\right) - 2\right)$$

After some algebra, it simplifies to

$$\frac{1}{2}\Delta_t^{-1}\frac{\partial \Delta_t}{\partial \psi}\frac{\partial \Delta_t}{\partial \omega_t} - \frac{\partial^2 \Delta_t}{\partial \omega_t \partial \psi} = -\frac{1}{\omega_t^2}\left(e^{2\psi} - e^{-2\psi}\right)\left(1 - \frac{1}{2\omega_t}\right)\left(1 + \Delta_t^{-1}\frac{2}{\omega_t}\right)$$

Therefore

$$\begin{aligned} \frac{\partial^2 R_t/R_t^*}{\partial \omega_t \partial \psi} &= \frac{1}{2}\left[\left(e^{\psi} - e^{-\psi}\right)\frac{1}{2\omega_t^2} + \frac{1}{2}\Delta_t^{-1/2}\frac{1}{\omega_t^2}\left(e^{2\psi} - e^{-2\psi}\right)\left(1 - \frac{1}{2\omega_t}\right)\left(1 + \Delta_t^{-1}\frac{2}{\omega_t}\right)\right] \\ &= \frac{1}{4\omega_t^2}\left(e^{\psi} - e^{-\psi}\right)\left[1 + \Delta_t^{-1/2}\left(e^{\psi} + e^{-\psi}\right)\left(1 - \frac{1}{2\omega_t}\right)\left(1 + \Delta_t^{-1}\frac{2}{\omega_t}\right)\right] \end{aligned}$$

It can be verified that the expression above is positive for all $0 \leq \omega_t \leq 1$ *and* $0 \leq \psi \leq 1$.

Hence

$$\frac{\partial^2 R_t/R_t^*}{\partial\omega_t\partial\psi} \geq 0 \qquad \textit{for} \quad 0 < \omega_t < 1 \quad \textit{and} \quad 0 \leq \psi \leq 1$$

with equality if and only if $\psi = 0$.

■

Lemma 5 (Range of R_t/R_t^*)
Under the assumed distribution of foreign inflation:

- *The relative interest rate R_t/R_t^* takes values in the following range*

$$\frac{2}{e^{\psi}+e^{-\psi}} \leq \frac{R_t}{R_t^*} \leq \frac{1}{2}\left(e^{\psi}+e^{-\psi}\right) \qquad \text{for} \quad 0<\omega_t<1 \quad \text{and} \quad \psi \geq 0$$

Proof (Lemma 5)
Recall that

$$\frac{R_t}{R_t^*} = \frac{1}{2}\left[\left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)+\Delta_t^{1/2}\right]$$

Also, recall that Lemma 2 implies that

$$\frac{\partial R_t/R_t^*}{\partial \omega_t} \geq 0 \qquad \text{for} \quad 0<\omega_t<1 \quad \text{and} \quad \psi \geq 0$$

Therefore, R_t/R_t^* must approach its infimum and supremum at $\omega_t = 0$ and $\omega_t = 1$ respectively.

Substituting Eq. (6) for Δ_t above, we can write

$$\frac{R_t}{R_t^*} = \frac{1}{2}\left[\left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)+\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(1-\frac{1}{2\omega_t}\right)^2+\frac{1}{\omega_t^2}\right)^{1/2}\right]$$

or

$$\frac{R_t}{R_t^*} = \frac{1}{2\omega_t}\left[\left(e^{\psi}+e^{-\psi}\right)\left(\omega_t-\frac{1}{2}\right)+\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(\omega_t-\frac{1}{2}\right)^2+1\right)^{1/2}\right] \tag{15}$$

Hence

$$\left.\frac{R_t}{R_t^*}\right|_{\omega_t=1} = \frac{1}{2}\left[\left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2}\right)+\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(1-\frac{1}{2}\right)^2+1\right)^{1/2}\right] = \frac{1}{2}\left(e^{\psi}+e^{-\psi}\right)$$

We can note from the expression above that R_t/R_t^* is undefined for $\omega_t = 0$, hence we need to find its limit as $\omega_t \to 0$.

Note that the limit of the square brackets in Eq. (15) above is

$$\lim_{\omega_t \to 0} \quad \left[\left(e^{\psi} + e^{-\psi}\right)\left(\omega_t - \frac{1}{2}\right) + \left(\left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)^2 + 1 \right)^{1/2} \right] = 0$$

Therefore, we have an indefinite fraction with $0/0$ *form. Therefore, we need to use L'Hopital's rule to find the limit of the fraction.*

The derivative of the expression in the square brackets of Eq. (15) is

$$\left(e^{\psi} + e^{-\psi}\right) + \left(\left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)^2 + 1 \right)^{-1/2} \left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)$$

Thus, in the limit, as $\omega_t \to 0$*, the expression in the square brackets of Eq. (15) is equal to*

$$\frac{4}{e^{\psi} + e^{-\psi}}$$

Hence

$$\lim_{\omega_t \to 0} \frac{R_t}{R_t^*} = \frac{2}{e^{\psi} + e^{-\psi}}$$

Therefore

$$\frac{2}{e^{\psi} + e^{-\psi}} \leq \frac{R_t}{R_t^*} \leq \frac{1}{2}\left(e^{\psi} + e^{-\psi}\right) \qquad \textit{for} \quad 0 < \omega_t < 1 \quad \textit{and} \quad \psi \geq 0$$

■

Lemma 6 (Range of the first derivatives of R_t/R_t^*)
Under the assumed distribution of foreign inflation:

- *The first derivative of R_t/R_t^* with respect to ψ takes values in the following ranges*

$$-2\frac{e^{\psi}-e^{-\psi}}{\left(e^{\psi}+e^{-\psi}\right)^2} < \frac{\partial R_t/R_t^*}{\partial \psi} \leq 0 \qquad \textit{for} \quad 0<\omega_t\leq 1/2 \quad \textit{and} \quad 0\leq\psi\leq 1$$

and

$$0 \leq \frac{\partial R_t/R_t^*}{\partial \psi} < \frac{1}{2}\left(e^{\psi}-e^{-\psi}\right) \qquad \textit{for} \quad 1/2\leq\omega_t< 1 \quad \textit{and} \quad 0\leq\psi\leq 1$$

Proof (Lemma 6)
Recall that

$$\frac{R_t}{R_t^*}=\frac{1}{2}\left[\left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)+\Delta_t^{1/2}\right]$$

and

$$\frac{\partial R_t/R_t^*}{\partial \psi}=\frac{1}{2}\left[\left(e^{\psi}-e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)\left(1+\Delta_t^{-1/2}\left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)\right)\right]$$

Recall that Lemma 2 implies that

$$\frac{\partial R_t/R_t^*}{\partial \psi}\leq 0 \qquad \textit{if} \quad 0<\omega_t\leq 1/2 \qquad \textit{and} \qquad \frac{\partial R_t/R_t^*}{\partial \psi}\geq 0 \qquad \textit{if} \quad 1/2\leq\omega_t<1$$

On the other hand, Lemma 4 implies that

$$\frac{\partial^2 R_t/R_t^*}{\partial \omega_t \partial \psi}\geq 0 \qquad \textit{for} \quad 0<\omega_t<1 \quad \textit{and} \quad 0\leq\psi\leq 1$$

Therefore, $\frac{\partial R_t/R_t^*}{\partial \psi}$ must approach its infimum and supremum at $\omega_t=0$ and $\omega_t=1$ respectively.

We can write

$$\frac{\partial R_t/R_t^*}{\partial \psi}=\frac{1}{2}\left[\left(e^{\psi}-e^{-\psi}\right)\left(\omega_t-\frac{1}{2}\right)\left(\frac{1}{\omega_t}+\Delta_t^{-1/2}\left(e^{\psi}+e^{-\psi}\right)\left(\frac{1}{\omega_t}-\frac{1}{2\omega_t^2}\right)\right)\right] \tag{16}$$

Hence, the upper bound for $\frac{\partial R_t/R_t^*}{\partial \psi}$ *is*

$$\left.\frac{\partial R_t/R_t^*}{\partial \psi}\right|_{\omega_t=1} = \frac{1}{4}\left[\left(e^{\psi}-e^{-\psi}\right)\left(1+\frac{1}{2}\left(\frac{1}{4}\left(e^{\psi}-e^{-\psi}\right)^2+1\right)^{-1/2}\left(e^{\psi}+e^{-\psi}\right)\right)\right]$$
$$= \frac{1}{2}\left(e^{\psi}-e^{-\psi}\right)$$

To find the lower bound for $\frac{\partial R_t/R_t^*}{\partial \psi}$*, we can note from* Eq. (16) *that* $\frac{\partial R_t/R_t^*}{\partial \psi}$ *is undefined for* $\omega_t = 0$*, hence we need to find its limit as* $\omega_t \to 0$*.*

The last term in brackets in Eq. (16) *is*

$$\frac{1}{\omega_t}+\Delta_t^{-1/2}\left(e^{\psi}+e^{-\psi}\right)\left(\frac{1}{\omega_t}-\frac{1}{2\omega_t^2}\right) = \frac{1}{\omega_t}+\frac{\left(e^{\psi}+e^{-\psi}\right)\left(\frac{1}{\omega_t}-\frac{1}{2\omega_t^2}\right)}{\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(1-\frac{1}{2\omega_t}\right)^2+\frac{1}{\omega_t^2}\right)^{1/2}} =$$
$$= \frac{\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(1-\frac{1}{2\omega_t}\right)^2+\frac{1}{\omega_t^2}\right)^{1/2}+\omega_t\left(e^{\psi}+e^{-\psi}\right)\left(\frac{1}{\omega_t}-\frac{1}{2\omega_t^2}\right)}{\omega_t\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(1-\frac{1}{2\omega_t}\right)^2+\frac{1}{\omega_t^2}\right)^{1/2}}$$

Rearranging and multiplying the numerator and denominator by ω_t *yields*

$$\frac{\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(1-\frac{1}{2\omega_t}\right)^2+\frac{1}{\omega_t^2}\right)^{1/2}+\left(e^{\psi}+e^{-\psi}\right)\left(1-\frac{1}{2\omega_t}\right)}{\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(\omega_t-\frac{1}{2}\right)^2+1\right)^{1/2}}$$
$$= \frac{\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(\omega_t-\frac{1}{2}\right)^2+1\right)^{1/2}+\left(e^{\psi}+e^{-\psi}\right)\left(\omega_t-\frac{1}{2}\right)}{\omega_t\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(\omega_t-\frac{1}{2}\right)^2+1\right)^{1/2}}$$

Therefore, we can write

$$\frac{\partial R_t/R_t^*}{\partial \psi} = \frac{1}{2}\left(e^{\psi}-e^{-\psi}\right)\frac{\left(\omega_t-\frac{1}{2}\right)\left(\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(\omega_t-\frac{1}{2}\right)^2+1\right)^{1/2}+\left(e^{\psi}+e^{-\psi}\right)\left(\omega_t-\frac{1}{2}\right)\right)}{\omega_t\left(\left(e^{\psi}-e^{-\psi}\right)^2\left(\omega_t-\frac{1}{2}\right)^2+1\right)^{1/2}} \tag{17}$$

Note that the limit of the numerator of the fraction above is

$$\begin{aligned}
&\lim_{\omega_t \to 0} \quad \left(\omega_t - \frac{1}{2}\right)\left(\left(\left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)^2 + 1\right)^{1/2} + \left(e^{\psi} + e^{-\psi}\right)\left(\omega_t - \frac{1}{2}\right)\right) \\
&\quad = -\frac{1}{2}\left(\left(\frac{1}{4}\left(e^{\psi} - e^{-\psi}\right)^2 + 1\right)^{1/2} - \frac{1}{2}\left(e^{\psi} + e^{-\psi}\right)\right) \\
&\quad = 0
\end{aligned}$$

while the limit of its denominator is

$$\lim_{\omega_t \to 0} \quad \omega_t \left(\left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)^2 + 1\right)^{1/2} = 0$$

Therefore, we have an indefinite fraction with 0/0 form. Therefore, we need to use L'Hopital's rule to find the limit of the fraction.

The derivative of the numerator is

$$\begin{aligned}
&\left(\left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)^2 + 1\right)^{1/2} + \left(e^{\psi} + e^{-\psi}\right)\left(\omega_t - \frac{1}{2}\right) + \\
&\left(\omega_t - \frac{1}{2}\right)\left(\left(\left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)^2 + 1\right)^{-1/2} \left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right) + \left(e^{\psi} + e^{-\psi}\right)\right)
\end{aligned}$$

On the other hand, the derivative of the denominator is

$$\left(\left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)^2 + 1\right)^{1/2} + \omega_t \left(\left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)^2 + 1\right)^{-1/2} \left(e^{\psi} - e^{-\psi}\right)^2 \left(\omega_t - \frac{1}{2}\right)$$

Thus, in the limit, as $\omega_t \to 0$, the numerator is equal to

$$\left(\frac{1}{4}\left(e^{\psi} - e^{-\psi}\right)^2 + 1\right)^{1/2} - \left(e^{\psi} + e^{-\psi}\right) + \frac{1}{4}\left(\frac{1}{4}\left(e^{\psi} - e^{-\psi}\right)^2 + 1\right)^{-1/2} \left(e^{\psi} - e^{-\psi}\right)^2$$

On the other hand, in the limit, as $\omega_t \to 0$, the denominator is equal to

$$\left(\frac{1}{4}\left(e^{\psi} - e^{-\psi}\right)^2 + 1\right)^{1/2}$$

Hence, by combining Eq. (17) with the limits obtained above, we get

$$\lim_{\omega_t \to 0} \frac{\partial R_t/R_t^*}{\partial \psi} = \frac{1}{2}\left(e^{\psi} - e^{-\psi}\right)\left[1 + \frac{\left(e^{\psi} - e^{-\psi}\right)^2}{\left(e^{\psi} - e^{-\psi}\right)^2 + 4} - 2\frac{e^{\psi} + e^{-\psi}}{\left(\left(e^{\psi} - e^{-\psi}\right)^2 + 4\right)^{1/2}}\right]$$

which, after some algebra, simplifies to[2]

$$\lim_{\omega_t \to 0} \frac{\partial R_t/R_t^*}{\partial \psi} = -2\frac{e^{\psi} - e^{-\psi}}{\left(e^{\psi} + e^{-\psi}\right)^2}$$

Therefore

$$-2\frac{e^{\psi} - e^{-\psi}}{\left(e^{\psi} + e^{-\psi}\right)^2} < \frac{\partial R_t/R_t^*}{\partial \psi} \leq 0 \qquad \textit{for} \quad 0 < \omega_t \leq 1/2 \quad \textit{and} \quad 0 \leq \psi \leq 1$$

and

$$0 \leq \frac{\partial R_t/R_t^*}{\partial \psi} < \frac{1}{2}\left(e^{\psi} - e^{-\psi}\right) \qquad \textit{for} \quad 1/2 \leq \omega_t < 1 \quad \textit{and} \quad 0 \leq \psi \leq 1$$

■

[2] Note, that the function above attains its minimum at $e^{\psi} - e^{-\psi} = 2$. This implies that $\psi = \log\left(1 + \sqrt{2}\right)$ and the minimum of the function at that point is $-\frac{1}{2}$.

Lemma 7 (Derivative of domestic debt)
The derivative of domestic debt level, D_t, with respect to the policy parameter, ϕ, can be expressed as

$$\frac{dD_t}{d\phi} = \frac{d\Lambda_t}{d\phi} D_{t-1} + \Lambda_t \frac{d\Lambda_{t-1}}{d\phi} D_{t-2} + \Lambda_t \Lambda_{t-1} \frac{d\Lambda_{t-2}}{d\phi} D_{t-3} + \ldots + \Lambda_t \Lambda_{t-1} \ldots \Lambda_2 \frac{d\Lambda_1}{d\phi} D_0$$

where $\Lambda_t \equiv \frac{R_{t-1}}{\Pi_t} - \tau$.

Proof (Lemma 7)
Recall that the law of motion for domestic debt is

$$D_t = \frac{R_{t-1}}{\Pi_t} D_{t-1} + G_t - T_t$$

Substituting $T_t = \tau D_{t-1}$ *we obtain*

$$D_t = G_t + \Lambda_t D_{t-1}$$

where $\Lambda_t \equiv \frac{R_{t-1}}{\Pi_t} - \tau$

Backwards substitutions for D_{t-i} *up to the initial level* D_0 *yield*

$$D_t = G_t + \Lambda_t G_{t-1} + \Lambda_t \Lambda_{t-1} G_{t-2} + \ldots + \Lambda_t \Lambda_{t-1} \ldots \Lambda_2 G_1 + \Lambda_t \Lambda_{t-1} \ldots \Lambda_1 D_0 \tag{18}$$

Then, noting that G_t *is exogenous, and taking the derivative of* D_t *with respect to* ϕ *yields*

$$\frac{dD_t}{d\phi} = \frac{d\Lambda_t}{d\phi} G_{t-1} + \frac{d}{d\phi}\left(\Lambda_t \Lambda_{t-1}\right) G_{t-2} + \ldots + \frac{d}{d\phi}\left(\Lambda_t \Lambda_{t-1} \ldots \Lambda_2\right) G_1 + \frac{d}{d\phi}\left(\Lambda_t \Lambda_{t-1} \ldots \Lambda_1\right) D_0$$

Evaluating the derivatives we obtain

$$\begin{aligned}
\frac{dD_t}{d\phi} = \frac{d\Lambda_t}{d\phi} G_{t-1} &+ \left(\frac{d\Lambda_t}{d\phi}\Lambda_{t-1} + \Lambda_t \frac{d\Lambda_{t-1}}{d\phi}\right) G_{t-2} \\
&+ \left(\frac{d\Lambda_t}{d\phi}\Lambda_{t-1}\Lambda_{t-2} + \Lambda_t \frac{d\Lambda_{t-1}}{d\phi}\Lambda_{t-2} + \Lambda_t \Lambda_{t-1}\frac{d\Lambda_{t-2}}{d\phi}\right) G_{t-3} \\
&+ \ldots \\
&+ \left(\frac{d\Lambda_t}{d\phi}\Lambda_{t-1}\Lambda_{t-2}\ldots\Lambda_2 + \Lambda_t \frac{d\Lambda_{t-1}}{d\phi}\Lambda_{t-2}\Lambda_{t-3}\ldots\Lambda_2 + \ldots + \Lambda_t \Lambda_{t-1}\ldots\Lambda_3 \frac{d\Lambda_2}{d\phi}\right) G_1 \Big] \\
&+ \left(\frac{d\Lambda_t}{d\phi}\Lambda_{t-1}\Lambda_{t-2}\ldots\Lambda_1 + \Lambda_t \frac{d\Lambda_{t-1}}{d\phi}\Lambda_{t-2}\Lambda_{t-3}\ldots\Lambda_1 + \ldots + \Lambda_t \Lambda_{t-1}\ldots\Lambda_2 \frac{d\Lambda_1}{d\phi}\right) D_0
\end{aligned}$$

which, after collecting terms, can be written as

$$
\begin{aligned}
\frac{dD_t}{d\phi} = & \left(\frac{d\Lambda_t}{d\phi} \left[G_{t-1} + \Lambda_{t-1} G_{t-2} + \ldots + \Lambda_{t-1}\Lambda_{t-2} \ldots \Lambda_2 G_1 + \Lambda_{t-1}\Lambda_{t-2} \ldots \Lambda_1 D_0 \right] \right) \\
& + \left(\Lambda_t \frac{d\Lambda_{t-1}}{d\phi} \left[G_{t-2} + \Lambda_{t-2} G_{t-3} + \ldots + \Lambda_{t-2}\Lambda_{t-3} \ldots \Lambda_2 G_1 + \Lambda_{t-2}\Lambda_{t-3} \ldots \Lambda_1 D_0 \right] \right) \\
& + \ldots \\
& + \left(\Lambda_t \Lambda_{t-1} \ldots \Lambda_4 \frac{d\Lambda_3}{d\phi} \left[G_2 + \Lambda_2 G_1 + \Lambda_2 \Lambda_1 D_0 \right] \right) \\
& + \left(\Lambda_t \Lambda_{t-1} \ldots \Lambda_3 \frac{d\Lambda_2}{d\phi} \left[G_1 + \Lambda_1 D_0 \right] \right) + \left(\Lambda_t \Lambda_{t-1} \ldots \Lambda_2 \frac{d\Lambda_1}{d\phi} D_0 \right)
\end{aligned}
$$

Therefore, substituting Eq. (18) for the terms in parentheses above, we obtain

$$
\frac{dD_t}{d\phi} = \frac{d\Lambda_t}{d\phi} D_{t-1} + \Lambda_t \frac{d\Lambda_{t-1}}{d\phi} D_{t-2} + \Lambda_t \Lambda_{t-1} \frac{d\Lambda_{t-2}}{d\phi} D_{t-3} + \ldots + \Lambda_t \Lambda_{t-1} \ldots \Lambda_2 \frac{d\Lambda_1}{d\phi} D_0
$$

■

Lemma 8 (Determinants of mean domestic debt)
The following statements hold for the mean domestic debt level, $E[D_t]$, i.e., the average domestic debt level across all generations.

- *Under an exchange rate peg, i.e., at $\phi = 1$, the mean domestic debt $E[D_t]$ is weakly increasing in ϕ:*

$$\left.\frac{dE[D_t]}{d\phi}\right|_{\phi=1} \geq 0$$

  with equality if and only if $\sigma_{\pi^*} = 0$.

- *Under inflation targeting, i.e., at $\phi = 0$, such that the economy is at its long-run equilibrium*

  - *mean domestic debt $E[D_t]$ is weakly increasing in ϕ if $0 < \overline{\omega} \leq 1/2$:*

$$\left.\frac{dE[D_t]}{d\phi}\right|_{\phi=0} \geq 0 \qquad \text{if} \quad 0 < \overline{\omega} \leq 1/2$$

  - *mean domestic debt $E[D_t]$ is weakly decreasing in ϕ if $1/2 \leq \overline{\omega} < 1$:*

$$\left.\frac{dE[D_t]}{d\phi}\right|_{\phi=0} \leq 0 \qquad \text{if} \quad 1/2 \leq \overline{\omega} < 1$$

  with equality if and only if $\overline{\omega} = 1/2$.

Proof (Lemma 8)
Recall from Lemma 7 that the derivative of domestic debt level with respect to the policy parameter can be expressed as

$$\frac{dD_t}{d\phi} = \frac{d\Lambda_t}{d\phi}D_{t-1} + \Lambda_t\frac{d\Lambda_{t-1}}{d\phi}D_{t-2} + \Lambda_t\Lambda_{t-1}\frac{d\Lambda_{t-2}}{d\phi}D_{t-3} + \ldots + \Lambda_t\Lambda_{t-1}\ldots\Lambda_2\frac{d\Lambda_1}{d\phi}D_0 \tag{19}$$

where $\Lambda_t \equiv \frac{R_{t-1}}{\Pi_t} - \tau$.

On the other hand, we may write

$$\frac{dE[D_t]}{d\phi} = E\left[\frac{dD_t}{d\phi}\right]$$

which by Eq. (19) becomes

$$\frac{dE\left[D_t\right]}{d\phi} = E\left[\frac{d\Lambda_t}{d\phi}D_{t-1}\right] + E\left[\Lambda_t\frac{d\Lambda_{t-1}}{d\phi}D_{t-2}\right] + E\left[\Lambda_t\Lambda_{t-1}\frac{d\Lambda_{t-2}}{d\phi}D_{t-3}\right] + \\ + \ldots + E\left[\Lambda_t\Lambda_{t-1}\ldots\Lambda_2\frac{d\Lambda_1}{d\phi}D_0\right] \tag{20}$$

Also, note that

$$\frac{d\Lambda_t}{d\phi} = \frac{dR_{t-1}}{d\phi}\frac{1}{\Pi_t} - \frac{R_{t-1}}{\Pi_t^2}\frac{d\Pi_t}{d\phi} = -\sigma_{\pi^*}R^*_{t-1}\frac{dR_{t-1}/R^*_{t-1}}{d\psi}\frac{1}{\Pi_t} - R_{t-1}\frac{\pi_t^*}{\Pi_t}$$

- *Under an exchange rate peg, we have $\phi = 1$ and $R_t = R_t^*$.*

  Therefore

$$\Lambda_t\big|_{\phi=1} = \frac{R^*_{t-1}}{\Pi^*_t} - \tau \qquad\qquad \left.\frac{d\Lambda_t}{d\phi}\right|_{\phi=1} = -R^*_{t-1}\frac{\pi^*_t}{\Pi^*_t}$$

  Note that since foreign interest rate is fixed, $R_t^* = R^*$, and foreign inflation π_t^* is assumed to be IID, then Λ_t and $\frac{d\Lambda_t}{d\phi}$ are IID, too.

  Therefore, Eq. (20) under an exchange rate peg becomes

$$\left.\frac{dE\left[D_t\right]}{d\phi}\right|_{\phi=1} = E\left[\frac{d\Lambda_t}{d\phi}\right]E\left[D_{t-1}\right] + E\left[\Lambda_t\right]E\left[\frac{d\Lambda_{t-1}}{d\phi}\right]E\left[D_{t-2}\right] + \\ + \ldots + E\left[\Lambda_t\right]E\left[\Lambda_{t-1}\right]\ldots E\left[\Lambda_2\right]E\left[\frac{d\Lambda_1}{d\phi}\right]D_0 \tag{21}$$

  On the other hand, note that

$$\left.E\left[\frac{d\Lambda_t}{d\phi}\right]\right|_{\phi=1} = \frac{\sigma_{\pi^*}}{2}R^*\left(e^{\sigma_{\pi^*}} - e^{-\sigma_{\pi^*}}\right) \geq 0$$

  Therefore, provided that $\Lambda_t > 0$ and $D_t > 0$ for all t, all terms in Eq. (21) are non-negative and we have

$$\left.\frac{dE\left[D_t\right]}{d\phi}\right|_{\phi=1} \geq 0$$

  with equality if and only if $\sigma_{\pi^*} = 0$.

- *Under inflation targeting, we have* $\phi = 0$ *and* $\pi_t = 0$ *and*

$$\left.\frac{d\Lambda_t}{d\phi}\right|_{\phi=0} = -\sigma_{\pi^*} R^*_{t-1} \frac{dR_{t-1}/R^*_{t-1}}{d\psi} - \pi^*_t R_{t-1}$$

Note that since under inflation targeting the economy is in the long-run equilibrium, then $R_t = \overline{R}$, $\Lambda_t = \overline{\Lambda}$ *and* $D_t = \overline{D}$ *are constant for all* t*. Also, since foreign interest rate is fixed,* $R^*_t = R^*$*, and foreign inflation* π^*_t *is assumed to be IID, then* $\frac{d\Lambda_t}{d\phi}$ *is IID, too.*

Therefore, Eq. (20) under inflation targeting becomes

$$\left.\frac{dE\left[D_t\right]}{d\phi}\right|_{\phi=0} = E\left[\frac{d\Lambda_t}{d\phi}\right]\overline{D} + \overline{\Lambda} E\left[\frac{d\Lambda_{t-1}}{d\phi}\right]\overline{D} + \ldots + \overline{\Lambda}^{t-1} E\left[\frac{d\Lambda_1}{d\phi}\right]\overline{D}$$

On the other hand, note that since $E\left[\pi^*_t\right] = 0$*, then*

$$\left. E\left[\frac{d\Lambda_t}{d\phi}\right]\right|_{\phi=0} = -\sigma_{\pi^*} R^* \frac{dR_{t-1}/R^*_{t-1}}{d\psi}$$

Also, recall that Lemma 2 implies that

$$\frac{\partial R_t/R^*_t}{\partial \psi} \leq 0 \qquad \text{if} \quad 0 < \omega_t \leq 1/2 \qquad \text{and} \qquad \frac{\partial R_t/R^*_t}{\partial \psi} \geq 0 \qquad \text{if} \quad 1/2 \leq \omega_t < 1$$

with equality if and only if $\omega_t = 1/2$*.*

Therefore, provided that $\overline{\Lambda} > 0$ *and* $\overline{D} > 0$*, then*

$$\left.\frac{dE\left[D_t\right]}{d\phi}\right|_{\phi=0} \geq 0 \qquad \text{if} \quad 0 < \overline{\omega} \leq 1/2$$

and

$$\left.\frac{dE\left[D_t\right]}{d\phi}\right|_{\phi=0} \leq 0 \qquad \text{if} \quad 1/2 \leq \overline{\omega} < 1$$

with equality if and only if $\overline{\omega} = 1/2$*.*

■

E Figures

In this section, I present additional figures complementing the analysis in the main text.

E.1 Baseline model

To illustrate the dynamics of the baseline model under different monetary policy strategies, I present simulation results for key model variables in Fig. 1. The simulations are conducted over 100 periods for three distinct monetary policy strategies: inflation targeting ($\phi = 0$), a hybrid regime ($\phi = 0.5$), and exchange rate peg ($\phi = 1$).

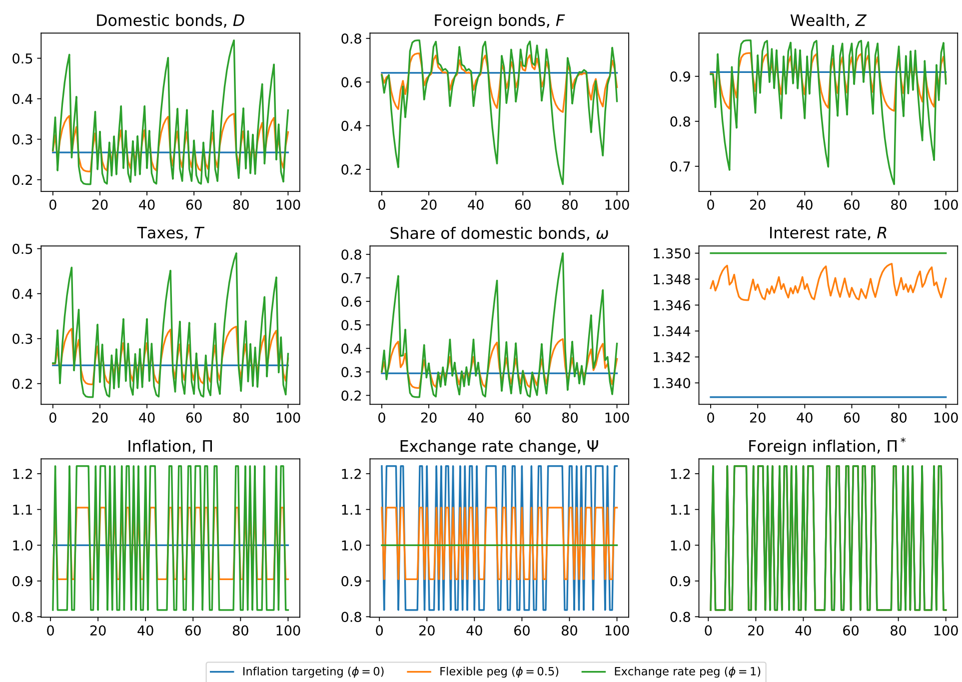

Figure 1: Model simulations under different monetary policy strategies

We can see that under inflation targeting, the economy in its long-run equilibrium with fixed prices. As a result, the economy is insulated from external shocks and all model variables remain constant over time.

As opposed to that, an exchange rate peg, the economy experiences fluctuations in response to foreign shocks. Notably, the volatility of key model variables increases with the degree of exchange rate stabilization, i.e., with higher values of ϕ.

On the other hand, the fluctuations in model variables exhibit an asymmetric pattern. Specifically, under the hybrid regime or an exchange rate peg, the domestic debt accumulates at a faster rate during periods of low domestic inflation compared to high inflation periods.

This asymmetry arises because low domestic inflation leads to high supply of government bonds, which in turn drives up the domestic interest rate and vice versa. Consequently, government debt accumulates more rapidly during low inflation periods, leading to a skewed distribution of model variables, as illustrated in Fig. 2.

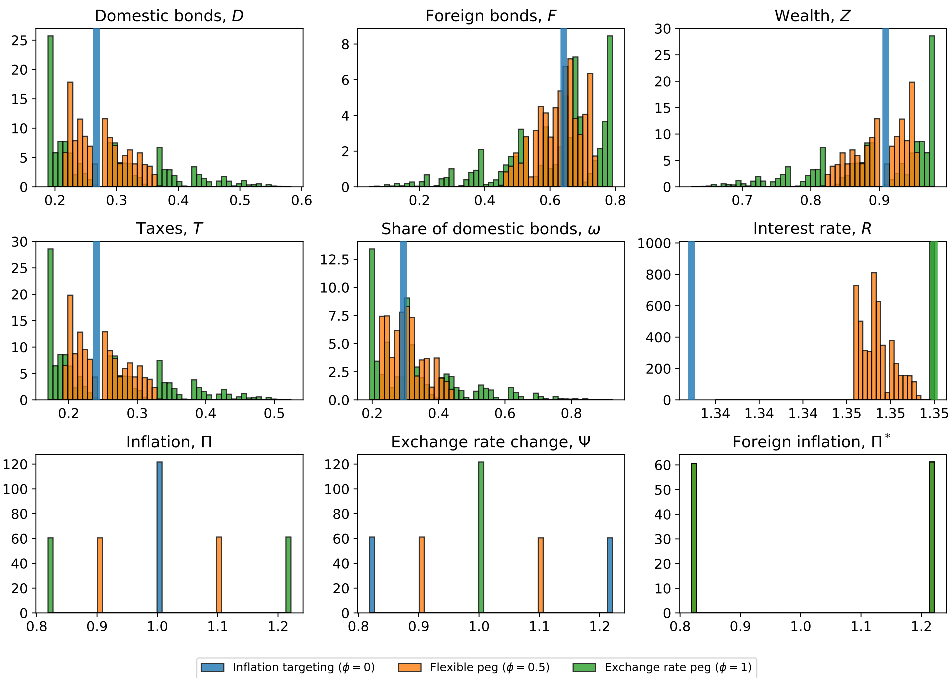

Figure 2: Distribution of model variables under different monetary policy strategies

The asymmetry in the distribution of model variables is a key feature of the model, as it highlights the non-linear effects of monetary policy on the economy. This non-linearity allows the central bank to affect the distribution of key economic variables. Thus, the monetary policy can drive mean values of model variables away from their steady-state levels by adjusting the monetary policy strategy, as shown in Fig. 3.

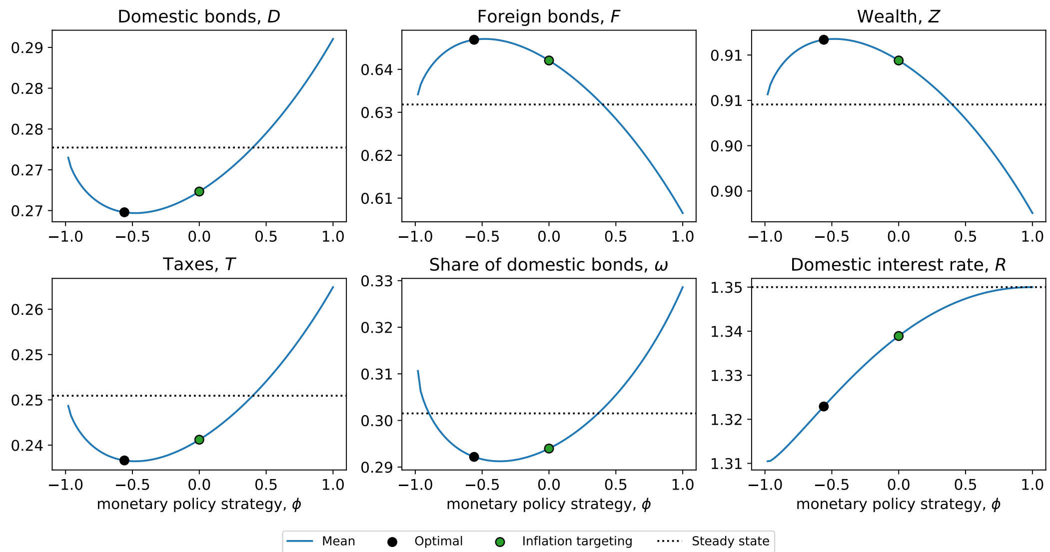

Figure 3: Mean values of model variables under different monetary policy strategies

As Fig. 3 shows, by choosing an optimal value of ϕ, the central bank can influence the mean values of key model variables, such as domestic bonds and interest rates. Specifically, lower values of ϕ lead to lower average domestic interest rates and consequently lower government debt levels under the optimal monetary policy strategy compared to inflation targeting and the exchange rate peg regimes.

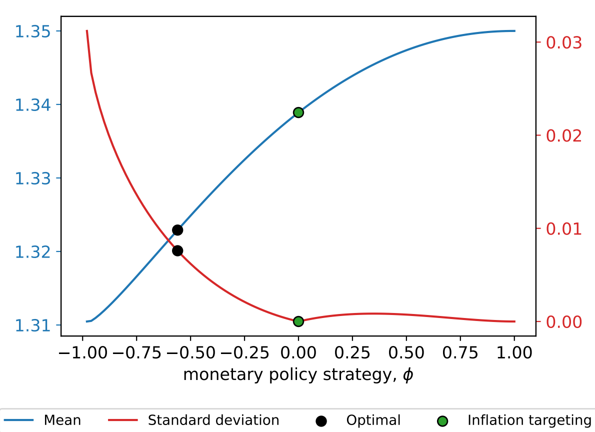

Figure 4: Mean and standard deviation of domestic interest rate under different monetary policy strategies

However, lower average domestic interest rates and government debt levels are achieved at the cost of higher volatility in these variables. This trade-off, illustrated in Fig. 4, highlights the central bank's challenge in balancing the three welfare effects discussed in the main text when determining the optimal monetary policy strategy. Specifically, as shown in Fig. 4, under the optimal policy the central bank achieves lower mean domestic interest rates compared to inflation targeting, but this comes with increased volatility in interest rates and government debt levels.

On the other hand, by choosing an optimal value of ϕ, the central bank can influence the portfolio choices of households across generations, thereby tilting the shares of domestic and foreign assets away from their steady-state levels. This effect may be seen in Fig. 5 and Fig. 6 which show how the mean debt level $E\left[D_t\right]$ and share of domestic assets $E\left[\omega_t\right]$ vary across generations with fiscal policy parameters under inflation targeting and optimal monetary policy strategies.

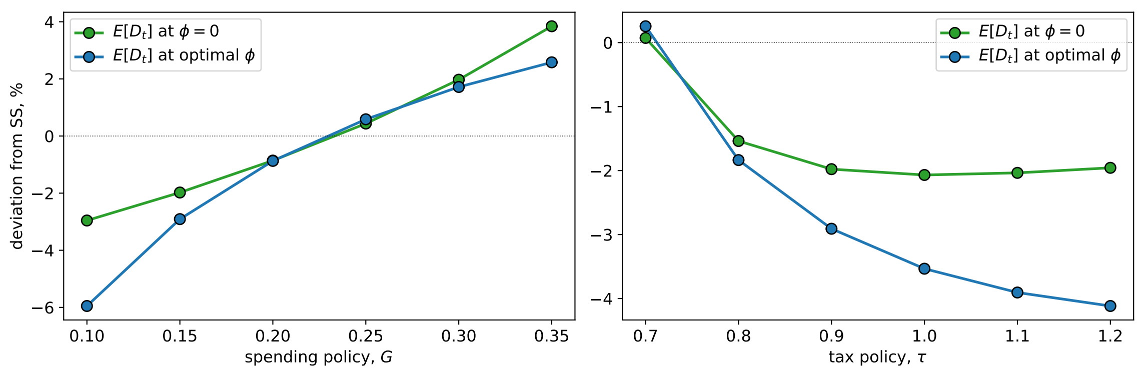

Figure 5: Mean government debt level under the optimal policy vs inflation targeting

As Fig. 5 shows, under inflation targeting households optimize their portfolios and drive government debt away from its steady-state level. In these case, government debt serves as a risk-free alternative to foreign assets and allows households to hedge their portfolios partially. However, as the central bank adopts the optimal policy, it improves the hedging properties of domestic assets and allows households to further exploit diversification benefits. The effect consequently leads to deviations in the share of domestic assets from their steady-state levels, as shown in Fig. 6.

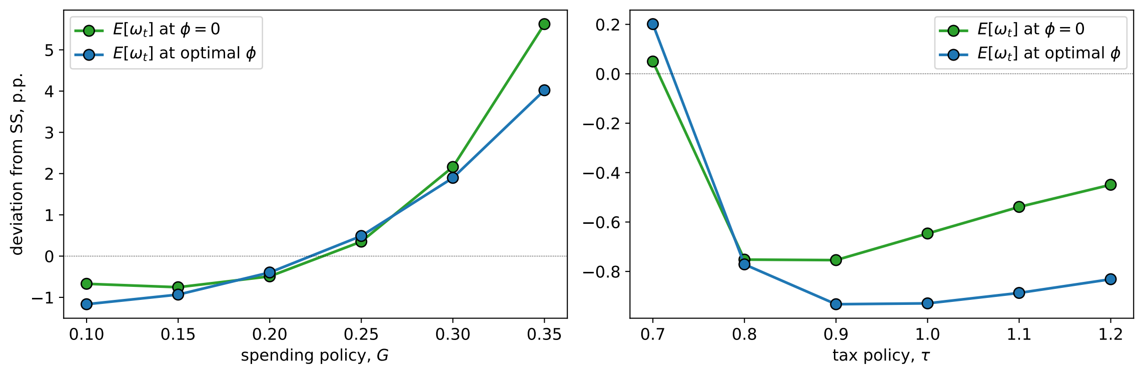

Figure 6: Mean share of domestic assets under different monetary policy strategies

Increased demand for domestic bonds under the optimal policy also leads to lower domestic interest rates, reducing borrowing costs for the government. Consequently, this results in a lower government debt levels compared to the inflation targeting regime for most of the fiscal policy configurations in Fig. 5. Therefore, the optimal monetary policy not only enhances household welfare through improved portfolio returns but also contributes to fiscal sustainability by lowering government debt levels.

E.2 Extensions

To study the implications of price rigidity on in the extended model with production, I present the optimal monetary policy strategy as function of the fiscal policy and country openness parameters under flexible prices in Fig. 7, which complements the analysis under sticky prices in the main text.

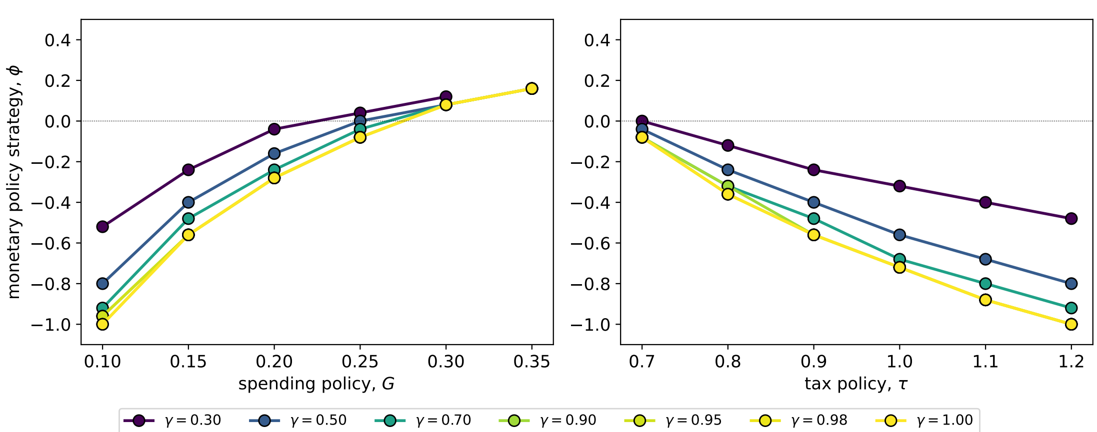

Figure 7: Optimal monetary policy strategy and country openness under flexible prices

Unlike the case with sticky prices, under flexible prices the optimal monetary policy strategy implies an optimal value of $\phi < 0$ for most combinations of fiscal policy and country openness parameters. This result suggests that in the absence of price rigidity, the central bank should lean towards negative domestic inflation co-movement with foreign inflation.

E.3 Robustness analysis

In this section, I study and present the implied optimal monetary policy strategy under different model specifications and shock processes to assess the robustness of the main results.

First, Fig. 8 illustrates how the optimal monetary policy strategy varies with the degree of government home bias. As the government home bias increases, fiscal policy returns the taxes collected from domestic households back to them in the form of social transfers.

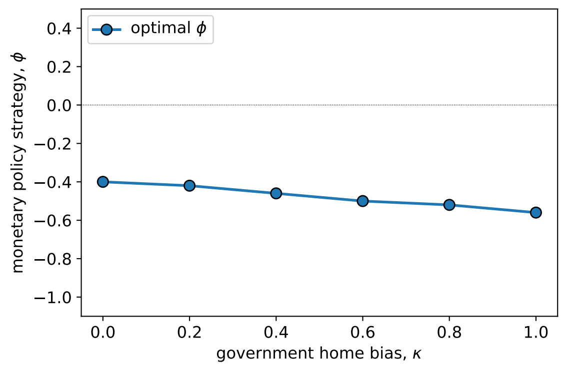

Figure 8: Optimal monetary policy strategy and government home bias

As Fig. 8 shows, higher government home bias leads to relatively lower optimal values of ϕ. This result indicates that the central bank should lean towards negative domestic inflation co-movement with foreign inflation when the government home bias is high.

Next, I study the implications of exogenous government spending and foreign interest rate shocks on the optimal monetary policy strategy. Specifically, the model is extended to include time-varying government spending and foreign interest rate both following an AR(1) process with persistence parameters of $\rho_g = \rho_r = 0.8$, and innovation standard deviations of $\sigma_g = \sigma_r = 0.03$, respectively.

Fig. 9 shows the optimal monetary policy strategy when the economy is subject to government spending and foreign interest rate shocks in addition to foreign inflation shocks.

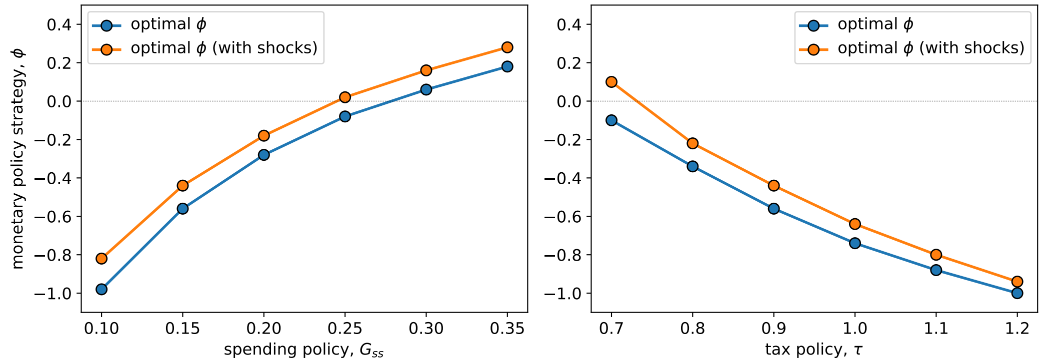

Figure 9: Optimal policy parameter ϕ with government spending and interest rate shocks

As Fig. 9 shows, the inclusion of government spending and foreign interest rate shocks does not qualitatively change the main results. The optimal monetary policy strategy still implies negative domestic inflation co-movement with foreign inflation for a wide range of fiscal policy parameters.

As an alternative welfare criterion, I also consider a discounted aggregate welfare measure for the central bank's optimization problem. Specifically, the central bank maximizes the discounted sum of expected utilities across all generations with a discount factor of $\beta = 1/R_{ss}$. Fig. 10 shows the optimal monetary policy strategy under the discounted welfare criterion.

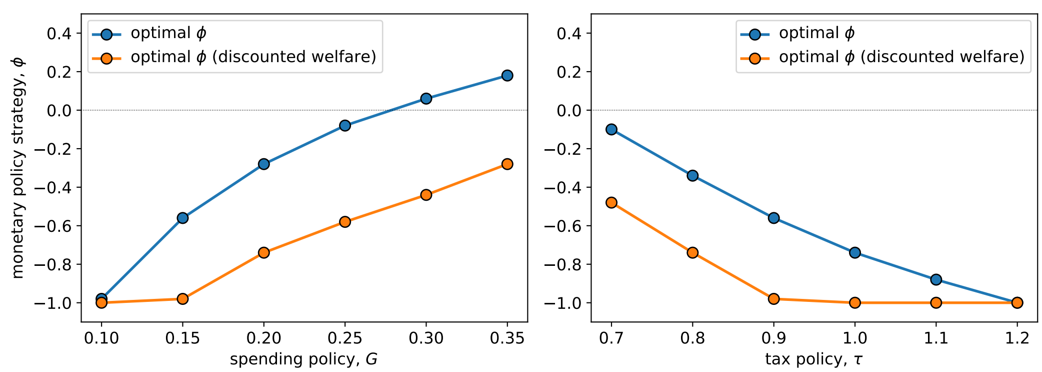

Figure 10: Optimal policy parameter ϕ with discounted welfare criterion

As Fig. 10 shows, the optimal policy regime is similar to the baseline case with average welfare maximization. In particular, the optimal policy parameter ϕ increases with government spending G and decreases with tax rate τ. However, under discounted utilities where future generations are weighted less, the long-run benefits of lower government debt levels and taxes are similarly discounted. As a result, the portfolio return effect becomes relatively more important, leading to significantly more negative optimal policy parameter ϕ compared to the baseline case.

Finally, Fig. 11 plots the optimal monetary policy strategies in two-state and multi-state economies to assess the robustness of the main results to the shock process specification.

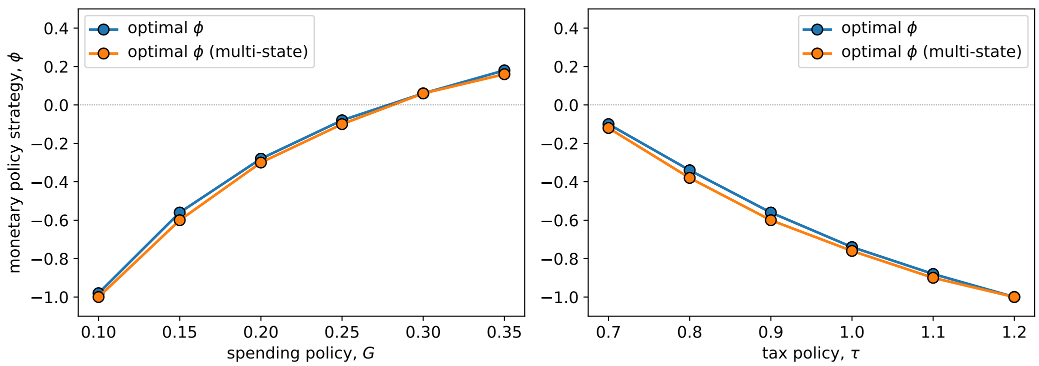

Figure 11: Optimal policy parameter ϕ in two-state and multi-state economies

We can see from Fig. 11 that the multi-state economy yields similar optimal monetary policy strategies as the baseline two-state economy. The optimal monetary policy strategy similarly implies negative domestic inflation co-movement with foreign inflation for a wide range of fiscal policy parameters.